Auction of Licenses to a Technology: What does the Experimental Evidence Say? *

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Abstract

We examine experimentally an auction model with externalities in which competing firms bid for licenses to a cost-reducing technology. Since winning bidders impose a negative externality on the losers, bids must account for both the value of winning the auction and the negative value of losing brought about by rivals reducing their costs. Experimental treatments differ in the severity of the negative externality (based on the substitutability of competitors' products), and the number of licenses being auctioned. We find that subjects underbid relative to theoretical benchmarks for auctions of one license, but overbid when two licenses are auctioned. Nevertheless, mean revenues in the experiment are consistent with the predicted revenues. However, there are some differences between the distributions of experimental and predicted revenues. We propose a possible explanation for these differences rooted in a simple bidding heuristic.

1 Introduction

Consider several competing firms with similar cost structures. A newly invented process innovation promises to reduce the firms' marginal costs, though the firms differ in the extent of this reduction. If the inventor elects to auction licenses to this technology, how do industry structure and auction design factors influence the auctioneer's revenue?

While several authors forward theoretical predictions about such auction markets (e.g., Jehiel, Moldovanu, and Stachetti, 1996 and 1999; Jehiel and Moldovanu, 2000; Das Varma, 2002 and 2003; Bagchi, 2005b), we provide theoretical benchmarks and examine experimentally two specific questions. First, as the industry becomes more competitive, perhaps through less differentiation of products, what happens to auction revenue? More intense competition depresses overall industry profitability but also implies that losers of the auction suffer a greater externality from winners' lower costs, perhaps increasing bids. Second, how does auction revenue change with the decision to offer multiple licenses for sale? Selling more licenses implies that more firms pay the auctioneer but also diminishes the competitive advantage of any single winner since others also enjoy reduced costs, potentially reducing bids. In all cases, participants' bids depend not only on the extent of cost savings they may anticipate from the new technology but also on their beliefs about the efficiency of rivals.

We develop a model of an inventor selling licenses for a cost-reducing technology using a uniform price auction. Each firm bids for at most one license; if k licenses are auctioned, the k firms with the highest bids win a license and pay an amount equal to the $(k+1)^{th}$ highest bid.[1] Prior to the auction, firms have identical costs though they differ in their abilities to implement the process improvement. Firms receive private signals as to the degree of cost savings they can realize by winning a license. In our experiment, three firms participate in an auction for either one or two licenses. Payoffs from the auction are derived from Cournot competition in a differentiated-goods industry with costs determined by the allocation of licenses resulting from the auction. We vary the level of product differentiation in the Cournot industry which alters the extent to which one firm's cost savings impacts the profitability of its rivals.

In the symmetric equilibrium of our model, each firm bids its "intrinsic value" (Das Varma, 2003; Bagchi, 2005b), the expected difference between its profit from winning a license and its profit from losing the license to a competing firm with the same signal. Thus, the equilibrium bid incorporates, in an additive fashion, the expected change in profit both from winning and from losing the auction. Relative to theoretical predictions, subjects undervalue the profit from winning though possibly compensate for this by adding a positive constant to bids. In auctions

[1] Acquiring one license gives a firm user rights to the cost-reducing technology. If a single firm was able to acquire additional licenses, these would not have any direct effect on the firm's cost structure, but would have a preemptive motive, foreclosing on the use of the license by a competitor. This is not our focus. For a theoretical treatment of such cases, see Bagchi (2005a).

of two licenses, subjects also overemphasize the profit from losing. That is, they overreact to the possibility of profit losses from not obtaining a license. The net effect of these factors is overbidding in auctions of two licenses and underbidding in auctions of one license. Nevertheless, mean revenues are mostly in line with theoretical predictions except in the cases of multiple licenses being sold in the presence of moderate externalities. Therefore, we conclude that when the products are moderately differentiated, an auction of two licenses performs better than what the model suggests; in the other cases, the observed revenues are close enough to the predicted revenues.

Although revenues are mostly in line with theoretical predictions, there are systematic departures between observed and predicted *distributions* of revenues. In the sale of one license, observed revenues show less dispersion than predicted, while in the sale of two licenses, the observed revenue distributions generally lie below the theoretically predicted distribution. We show that a simple bidding heuristic can explain both anomalies. When bidding, a subject does not know the signals of other firms, which represent their level of cost savings upon acquiring a license. In the absence of this information, it is possible that subjects do not undertake the rather arduous task of calculating equilibrium expectations conditional on their own signals but instead simply assume (or act as if they assume) that the winners' private information is equal to the unconditional expected value of the signal. Predicted revenues for the auctioneer when each firm bids according to this simple heuristic are consistent with our data. On the whole, subjects appear to react to externalities in the manner that we predict and discrepancies between the experimental data and our predictions can be explained by the fact that subjects do not correctly estimate the signals of competitors who win a license. In real world auctions, firms are likely to form better estimates of competitors' signals than do our subjects, leading revenues to approximate our predictions more closely. However, for auctions in which bidders are not likely to show great sophistication, our heuristic provides a simple revenue benchmark. We show that both bidding frameworks – in line with the equilibrium or the heuristic – generally lead to the same guidance as to the optimal number of licenses to auction.

Markets for a single license with private information have recently received considerable attention (Jehiel, Moldovanu, and Stachetti, 1996 and 1999; Moldovanu and Sela, 2003; Katzman and Rhodes-Kropf, 2002; Goeree, 2003), including the analysis by Jehiel and Moldovanu (2000) of a second price auction and Das Varma (2003) of a first price auction. Several papers have considered the sale of multiple licenses. Katz and Shapiro (1986) and Hoppe, Jehiel, and Moldovanu (forthcoming) assume that the signal of a firm is publicly observable. Jehiel and Moldovanu (2001, 2004) demonstrate the impossibility of implementing efficient allocations when signals are multidimensional. We consider private, unidimensional signals representing the realized cost savings if a license is acquired, allowing for equilibrium characterization. Dana (1994) and Schmitz (2002) also consider the problem of auctioning production rights but assume that a firm that does not acquire a license earns zero profits, while we, following Bagchi (2005b), allow a losing firm's profits to decrease as a result of the cost savings realized by license-acquiring competitors.

Several papers examine experimentally auctions with interdependent valuations. Kirchkamp and Moldovanu (2004) consider auctions in which the winner's payoff depends on the private information of another, specific bidder. Only one object is sold, however, and winners do not impose an externality on losing bidders. Goeree, Offerman, and Sloof (2004) examine bidding when new entrants impose a negative externality on existing market participants. In multi-unit auctions, the authors find that the existence of externalities does not eliminate strategic demand reduction (Alsemgeest, Noussair, and Olson, 1998; List and Lucking-Reiley, 2000) when subjects can bid for multiple licenses. To our knowledge, Goeree, Offerman, and Sloof (2004) is the only other manuscript to consider auctions in which negative externalities impact both winners and losers.

2 Theoretical Considerations

2.1 Model

The model follows Bagchi (2005b) which extends the previous literature on single-object auctions with externalities to the case of multiple licenses. Notable papers on license auctions are Jehiel and Moldovanu (2000) that analyzes a second price auction and Das Varma (2003) that analyzes a first price auction of licenses when a winner signals her type through her bid. Consider an industry with n competing firms. The profit of firm i, $\pi(c_i; c_{-i}, \xi)$, depends on its own marginal cost of production, c_i, the vector of rivals' costs, c_{-i}, and a parameter, ξ, representing the the strength of externalities or the degree to which one firm's cost savings imposes an externality on its rivals. It may be convenient to interpret ξ as the degree of product substitutability. When ξ is large, firms are selling similar products leading to more intense competition, while a low value of ξ implies that competition among rivals is low. Firm i's profit is (i) decreasing in its own costs ($\frac{\partial\pi(c_i;c_{-i},\xi)}{\partial c_i} < 0$), (ii) increasing in others' marginal cost ($\frac{\partial\pi(c_i;c_{-i},\xi)}{\partial c_j} > 0, j \neq i$), and (iii) the effect of a change in a competitor's cost on i's profits is increasing in ξ, the externality parameter ($\frac{\partial^2\pi(c_i;c_{-i},\xi)}{\partial\xi\partial c_j} > 0, j \neq i$). Assumption (ii) suggests that a decrease in costs imposes a negative externality on one's competitors. Assumption (iii) allows us to vary the strength of this externality.

Initially, all firms have identical marginal costs, $\overline{c}$, perhaps a result of access to an existing publicly available technology or convergence of industry practice. An independent inventor develops a new cost-reducing technology which benefits firms differentially. In particular, each firm i receives a private signal $\theta_i \in [0, \overline{c}]$ drawn independently from the distribution function $F(\theta)$. These signals represent the cost savings realized by the firm if it were to employ the new technology, reducing its marginal cost to $c_i = \overline{c} - \theta_i$. Let $\theta_{(k)}^{n-1}$ be the k^{th} highest signal among firm i's competitors, so that

$$\theta_{(1)}^{n-1} \geq \theta_{(2)}^{n-1} \geq \ldots \geq \theta_{(n-1)}^{n-1}. \tag{1}$$

It will be convenient to express profit as a function of the firms' signals. Suppose that the k firms who could achieve the greatest cost savings (have the highest values of θ_i) obtain licenses to the new production technology through an auction. If firm i is among the winners, its profit is given by:

$$\Pi\left(\theta_i; \theta_{(1)}^{n-1}, \ldots, \theta_{(k-1)}^{n-1}, \xi\right) \equiv \pi\left(\overline{c} - \theta_i; \overline{c} - \theta_{(1)}^{n-1}, \ldots, \overline{c} - \theta_{(k-1)}^{n-1}, \overline{c}_{n-k}, \xi\right) \tag{2}$$

where $\overline{c}_{n-k}$ is a vector of dimension $n-k$ whose components equal $\overline{c}$.

Analogously, when firm i is not among the k firms with the highest values of θ, the firm continues to employ the old technology and earns profits of:

$$\Pi\left(0;\theta_{(1)}^{n-1},\ldots,\theta_{(k)}^{n-1},\xi\right)\equiv\pi\left(\overline{c};\overline{c}-\theta_{(1)}^{n-1},\ldots,\overline{c}-\theta_{(k)}^{n-1},\overline{c}_{n-1-k},\xi\right). \quad (3)$$

Lastly, we consider the profit prior to any firm acquiring a license. Each firm has a cost of $\overline{c}$, or equivalently, a cost saving of 0, with profit given by

$$\Pi\left(0;\xi\right)\equiv\pi\left(\overline{c};\overline{c}_{n-1},\xi\right). \quad (4)$$

The inventor of the technology auctions $k<n$ licenses, with firms submitting bids for a single license. Though the model may be generalized to most common auction formats, we consider a uniform-price auction in which each of the k highest bidders wins a license and pays an amount equal to the $(k+1)^{th}$ highest bid. We restrict our attention to the increasing symmetric equilibrium, which implies that the firms with the greatest cost savings (signals) win the auction.

2.2 Equilibrium

Firms that win a license enjoy increased profits while losing firms suffer decreased profits when competitors reduce costs. In our experiment, we study the role of each of these effects – the profit from winning and from losing – in determining subjects' bids. Define the *winning value*, the change in profit accruing to a winner of the auction, as

$$W_k\left(\theta_i,\xi\right)=E\left[\Pi\left(\theta_i;\theta_{(1)}^{n-1},\ldots,\theta_{(k-1)}^{n-1},\xi\right)|\theta_{(k)}^{n-1}=\theta_i\right]-\Pi\left(0;\xi\right) \quad (5)$$

The winning value is the expected increase in profit from obtaining a license. It is the difference between firm i's profit when winning the auction and its profit when every firm uses the old technology. Analogously, the *losing value* is given by

$$L_k\left(\theta_i,\xi\right)=E\left[\Pi\left(0;\theta_{(1)}^{n-1},\ldots,\theta_{(k)}^{n-1},\xi\right)|\theta_{(k)}^{n-1}=\theta_i\right]-\Pi\left(0;\xi\right) \quad (6)$$

and is the difference between firm i's profits from losing the auction when the marginal winner has the same signal as firm i and firm i's profits when all firms use the old technology.

For concreteness, consider the bidder's problem in the absence of externalities, as would occur if each firm were a monopolist in its market. The winning value would simply reflect the profit gain from lowering costs. The losing value would equal zero since other firms' costs do not enter into a monopolist's profit function. A private value auction with *i.i.d* valuations would ensue. In the presence of externalities, bidders must account for the reduction in profit that results from losing the auction and the impact of other winners if more than one license is sold. We define the *intrinsic value*, $V_k\left(\theta_i,\xi\right)$, as the difference between the winning value and the losing value for firm i when k

licenses are auctioned.[2]

$$\begin{aligned} V_k(\theta_i,\xi) &\equiv W_k(\theta_i,\xi) - L_k(\theta_i,\xi). \quad (7)\\ &= E\left[\Pi\left(\theta_i;\theta_{(1)}^{n-1},\ldots,\theta_{(k-1)}^{n-1},\xi\right) - \Pi\left(0;\theta_{(1)}^{n-1},\ldots,\theta_{(k)}^{n-1},\xi\right)|\theta_{(k)}^{n-1}=\theta_i\right]. \end{aligned}$$

The intrinsic value represents the expected difference in a firm's profit between winning a license and losing a license when the marginal winner's signal is θ_i. The following proposition characterizes the equilibrium of the auction.

Proposition 1 *In the unique increasing symmetric equilibrium of a uniform price auction of $k < n$ licenses, the bid of firm i is given by $b_k(\theta_i;\xi) = V_k(\theta_i,\xi)$.*

Proof. See the Appendix. ■

Since bids equal intrinsic values in equilibrium, expected revenue is characterized by the auctioneer collecting k payments each equal to the $(k+1)^{th}$ order statistic of intrinsic values.

Corollary 1 *The ex-ante expected revenue for the auctioneer of $k < n$ licenses in a uniform price auction is given by*

$$R_k(\xi) = kE\left[V_k\left(\theta_{(k+1)}^{n},\xi\right)\right] \quad (8)$$

Very little insight on the revenue effects of these auctions can be gleaned directly from this equation. First, increasing the externality parameter, ξ, may increase or decrease revenues, depending on the profit function. Greater competition generally decreases the value from winning the auction but also decreases a loser's profit, leading to ambiguous effects in the aggregate. Second, an increase in the number of licenses auctioned likewise may increase or decrease auction revenue. The effect depends on the relative changes in winning value and losing value caused by offering more units and on the properties of the order statistics of signals induced by the distribution $F(\theta)$. Lastly, revenue effects need not be monotonic in either the number of licenses sold, k, or the externality parameter, ξ. Thus, for some profit specifications, selling to a very competitive industry may result in higher profits than selling to a less competitive one, while the reverse can hold for other specifications. The experimental treatments we consider exhibit several of these features.

3 Experimental Design

We now describe the specific form of the model we use in the experiments. Three firms with differentiated products compete in quantities *a la* Cournot. The inverse demand function for firm i is given by

$$p_i = 300 - q_i - \xi\sum_{j\neq i} q_j\ , \quad (9)$$

[2] Das Varma (2003) analyzes a first price auction of a single license in which firms signal their types through their bids. One component of the bidding function in his model is a function of the intrinsic value. Bagchi (2005b) generalizes the intrinsic value for auctions of multiple licenses. The expression shares similarities with the equilibrium bid for a common value auction derived in Milgrom and Weber (1982).

where $\xi \in [0,1]$ captures the level of product differentiation. We consider three cases:

(i). $\xi = 0$ (monopoly) in which competitors' quantities do not influence own price,

(ii). $\xi = 1/2$ (differentiation) in which firms produce imperfect substitutes, and

(iii). $\xi = 1$ (homogeneity) which is the case of identical products.

Prior to the auction, each firm has a marginal cost of $\overline{c} = 100$. Auctions are either for one license ($k = 1$) or two licenses ($k = 2$). Each firm receives a private signal, θ_i, distributed *i.i.d.* uniform on $[0, 100]$, representing the cost savings resulting from winning a license.

For any outcome of the auction, the resulting change in profits for subject i is given by:

$$\Delta\Pi\left(\theta_i;\theta_j,\theta_k\right) = \left[\frac{200\left(2-\xi\right)+\left(2+\xi\right)I_i\theta_i-\xi\left(I_j\theta_j+I_k\theta_k\right)}{2\left(1+\xi\right)\left(2-\xi\right)}\right]^2-\left[\frac{100}{1+\xi}\right]^2 \tag{10}$$

where the first term is the post-auction profit, the second term is the pre-auction profit when each firm has a cost of 100, and I_l is an indicator function taking the value of 1 if player l is a winner of the auction and zero otherwise. Next, we derive predictions for equilibrium bids and revenues.

Proposition 2 *Equilibrium bids and resulting expected revenues for $k \in \{1,2\}$ are given by:*

$$b_1\left(\theta_i;\xi\right) = V_1\left(\theta_i;\xi\right) = \frac{\left[200\left(2-\xi\right)+\theta_i\right]\theta_i}{\left(1+\xi\right)\left(2-\xi\right)^2} \tag{11}$$

$$b_2\left(\theta_i;\xi\right) = V_2\left(\theta_i;\xi\right) = \frac{\left[100\left(8-5\xi\right)+\left(2-\xi\right)\theta_i\right]\theta_i}{2\left(1+\xi\right)\left(2-\xi\right)^2} \tag{12}$$

$$R_1(\xi) = \frac{10000\left(2.3-\xi\right)}{(1+\xi)(2-\xi)^2} \tag{13}$$

$$R_2(\xi) = \frac{10000\left(2.2-1.35\xi\right)}{(1+\xi)(2-\xi)^2} \tag{14}$$

In keeping with our intuition about Cournot competition, it can be verified that total industry profits decline as products become more substitutable (as ξ increases) for any profile of costs. However, declining industry profitability need not imply that the auctioneer's profit is decreasing. This specification of profits has four revenue implications of interest, reflected in Table 1. First, when selling a single license ($k = 1$), a perfectly homogeneous product market yields greater profit for the auctioneer than a market of monopolists. Second, the opposite result obtains for the sale of two licenses ($k = 2$). Third, in the single license case ($k = 1$), the intermediate case of $\xi = 0.5$ results in lower revenue than either polar case of $\xi = 0$ or $\xi = 1$, implying that revenues need not be monotonic in ξ. Finally, for the chosen parameters, auctioning one license generates more revenue than auctioning two licenses. We wish to examine how closely these revenue predictions match experimental data.

	$\xi = 0$ monopoly (no externalities)	$\xi = 0.5$ differentiation (weak externalities)	$\xi = 1$ homogeneity (strong externalities)
$k = 1$	5750	5333	6500
$k = 2$	5500	4519	4250

Table 1: Predicted revenue in experimental treatments

4 Experimental Method

Our subject population was comprised of 78 students at Vanderbilt University. Because the focus of the present study is on how subjects react to externalities, traditional overbidding observed in experiments of uniform-price auctions (e.g., Kagel, Harstad, and Levin, 1987; Kagel and Levin, 1993) would confound data interpretation. To avoid this, most subjects (92%) were M.B.A. students who had completed introductory lectures on the theory of auctions. Specifically, subjects had learned of the dominant strategy in second price auctions to bid one's value in private value settings, and had participated in several high-stakes experiments (for exemption from a final exam) as part of a course in game theory. No treatment of externalities was included in course materials. While there is evidence that experience in previous economics experiments tends to improve bidding behavior (Harstad, 2000), the results we describe in our monopoly treatments – which are equivalent to private value auctions – suggest that formal instruction is also likely to improve bidding behavior (McCabe and Smith, 2000).

Each subject participated in a series of either second price auctions for one license ($k = 1$) or third price auctions for two licenses ($k = 2$). Subjects bid in a total of 15 auctions, five at each of three values of the substitution parameter, $\xi \in \{0, 0.5, 1\}$. Subjects were told that in each auction, they would be randomly matched with two other participants whose identities would not be revealed.

Prior to each series of five auctions (for a specific value of ξ), subjects were shown the relevant payoff function (Equation 10 for the specific value of k and ξ) and were provided with tables that provided numerical values of this profit under various scenarios. In every auction, each subject received a signal from the uniform distribution on $[0, 100]$. For a specific signal in a particular auction, subjects were presented with tables of payoffs accruing to both a winner and loser of the auction for various signals of their competitors. Prior to bidding in each series of five auctions, subjects worked through an example and had to correctly calculate the resulting profits in two scenarios (winning and losing) to proceed. The majority of subjects successfully completed each of these tests on the first try. The average number of attempts on each test was 1.39. Subjects did not observe the outcome of any auction until the conclusion of the experiment, which limits intra-game learning and path dependency of wealth effects.

Subjects received a participation fee of \$20 to which winnings were added and losses deducted. Payoffs were denoted in points with 1000 points convertible into one dollar. These payoffs were governed by Equation 10. A winner of an auction earns the change in profit due to her decreased cost and pays the resulting price (second or third highest bid). A subject who loses an auction suffers lower profits due to others' cost reductions (except in the no-externality, monopoly case, when $\xi = 0$). Three subjects went broke, losing the entire participation fee during the experiment. These subjects were dropped from the sample for the purpose of the analysis.[3] The treatments and number of observations in each is reported in Table 2.

The experiment required an average of 29 minutes to complete and subjects earned an average of \$33. In actuality, all subjects were paid a minimum of \$15 even if their net earnings were lower than this amount, though they were not informed of this prior to the experiment.[4]

	monopoly ($\xi = 0$) 5 bids per subject	differentiation ($\xi = 0.5$) 5 bids per subject	homogeneity ($\xi = 1$) 5 bids per subject
one license ($k = 1$) $N = 39$	180 bids 60 auctions	180 bids 60 auctions	180 bids 60 auctions
two licenses ($k = 2$) $N = 39$	195 bids 65 auctions	195 bids 65 auctions	195 bids 65 auctions

Table 2: Experimental treatments

While subjects were free to bid any amount, we compute the maximum value a license could possibly hold for a participant. As is common in experiments with second price auctions, several subjects bid unreasonably high amounts which would significantly skew the analysis. For the purpose of analysis, subjects' bids were censored from above at the maximum possible difference in value between winning and losing the auction.[5] Any bid above this value is weakly dominated by bidding this value. Specifically, the most value one can derive from winning an auction occurs when the subject has the maximum signal of 100 and, if $k = 2$, when the other winner has a signal of 0. The worst loss can occur when a subject loses the auction and each winner has the maximum signal of 100. We censor at the difference between the best gain and worse loss. Thus, for $k = 1$, bids above 12,500, 11,852, and 15,000 were censored for $\xi = 0$, 0.5, and 1. Similarly, for $k = 2$, bids were censored at 12,500, 10,371, and 10,000. This censoring affected 2.4% of bids in the $\xi = 0$ condition, 9.1% when $\xi = 0.5$, and 8.3% when $\xi = 1$.

[3] All three subjects participated in the $k = 1$ treatment, and generally bid the same (unusually large) amounts in each auction regardless of their signals. Their inclusion in the data analysis increases parameter values in the $k = 1$ treatment.

[4] Keeping MBA subjects happy is a secondary, though institutionally-imposed, concern.

[5] Methods of dealing with severe overbidding above any reasonably obtainable value include prohibiting subjects from bidding above the maximum value (e.g., Mares and Shor, 2004), cautioning subjects not to do so (e.g., Kagel and Levin, 2001), or simply censoring such bids by setting them equal to the maximum possible valuation prior to data analysis (e.g., Kagel and Levin, 2004). Since the maximum intrinsic value is a more complicated object than just the maximum draw, censoring after the experiment seems the easiest approach as it requires no further explanation to subjects.

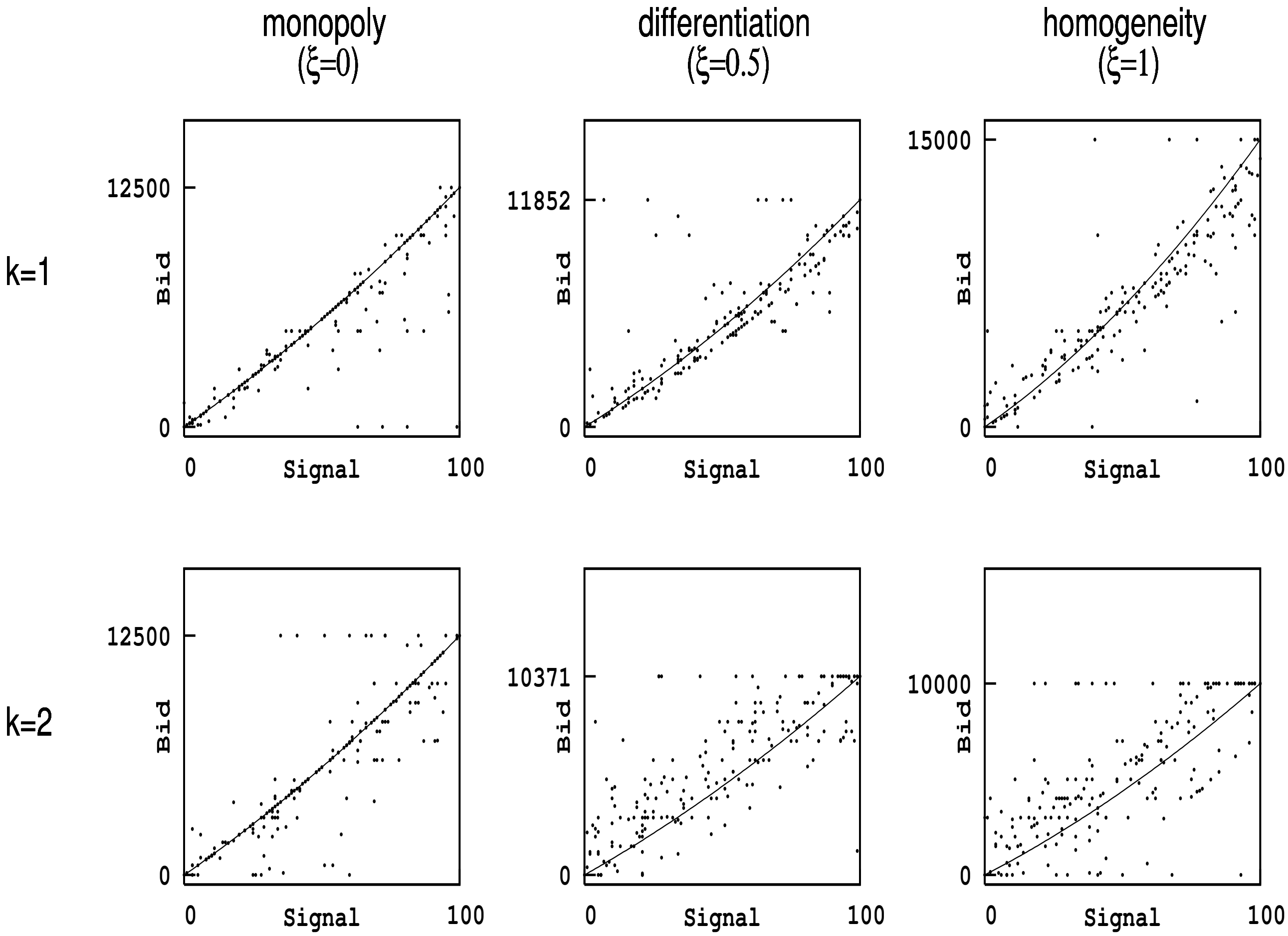

Figure 1: Observed versus predicted bids.

5 Results

5.1 Summary Statistics of Bids

We begin by examining raw data on bids. Plots suggest a strong co-movement between observed and equilibrium bids (Figure 1). In both monopoly cases, in which no externalities exist, most bids are at or slightly below the intrinsic value. When externalities are introduced, bids appear more dispersed and exhibit a higher incidence of values at the censored upper bound of permissible bids.

		Observed (std. dev.)	Predicted† (std. dev)	p-value ‡ $H_1 : obs \neq pred$
k=1	monopoly	5333	5876	< 0.01
$N = 180$	($\xi = 0$)	(3550)	(3676)	
	differentiation	5619	5558	0.66
	($\xi = 0.5$)	(3170)	(3237)	
	homogeneity	6558	6975	0.01
	($\xi = 1$)	(3997)	(4392)	
k=2	monopoly	5947	6090	0.28
$N = 195$	($\xi = 0$)	(3740)	(3388)	
	differentiation	5646	4678	< 0.01
	($\xi = 0.5$)	(3218)	(3151)	
	homogeneity	5172	4434	< 0.01
	($\xi = 1$)	(3329)	(2940)	

†Mean and standard deviation of intrinsic values evaluated at subjects' signals.
‡Results of matched pair t-test determining whether the distribution of differences between bids and corresponding intrinsic values is significant.

Table 3: Mean and Standard Deviation of the Bids

We do not find any evidence of overbidding in the monopoly cases. In the aggregate, subjects underbid relative to equilibrium predictions in an auction of one license and conform to equilibrium predictions in an auction of two licenses (Table 3). This departure from previous experiments lends support for our use of "sophisticated" bidders with familiarity of uniform-price auctions and allows us to conclude with some confidence that any departures from equilibrium in the presence of externalities (especially overbidding) are due to the externalities themselves. In the presence of externalities, we find evidence of overbidding only for auctions of two licenses.

Result 1 *Bids exceed theoretical predictions when two licenses are auctioned in the presence of externalities. In all other cases, subjects either underbid or bid in line with theoretical predictions.*

In the sale of one license, we expect bids to first decrease and then increase as the negative externalities become more prominent. Comparing bidding in the monopoly case ($\xi = 0$, no externalities) with the differentiated products case ($\xi = 0.5$), average bids actually increase, though not significantly ($p = .21$ one-tailed). As ξ increases from 0.5 to 1, bids increase ($p < .01$) in accord with theoretical predictions. When two licences are sold, consistent with theoretical predictions, subjects bid less when the level of competition increases. However, on average, this decline is not as dramatic as theoretically predicted. For our data, the change in mean bid is not significant ($p = .20$, one-tailed) when ξ increases from 0 to 0.5 and only mildly significant ($p = .08$) when ξ increases from 0.5 to 1.

Result 2 *Overall, bids appear to increase with the level of competition in the case of one license and to decrease (though by less than predicted) in the case of two licenses.*

In the presence of externalities, our subjects do not exceed theoretically predicted bids in an auction of one license but do overbid in auctions of two licenses. Before examining the implication of these behaviors on revenue, the next two subsections consider factors that may have contributed to the bidding patterns. We examine whether subjects properly account for both the value of winning and the value of losing a license in their bids. As a preview of the results, bids are on the whole in line with theoretical predictions. However, a simple heuristic model that does not require subjects to calculate conditional expectations about rivals' signals also conforms to observed bids.

5.2 Auctions of One License

We analyze whether subjects bid in accordance with theoretical predictions. Because we observe multiple bids for each subject, these 15 observations may exhibit similar subject-specific idiosyncracies and may not be considered independent. We also must account for the fact that bids are censored from above, and the points at which they are censored differ across treatments. We estimate a series of fixed-effects censored normal regressions of the following form:

$$bid_{ij} = \alpha_i + \beta X_{ij} + \epsilon_{ij},$$

where i indexes the subject and j indexes each of the 15 bids placed by that subject. The variable α_i captures person-specific fixed effects and X_{ij} is the matrix of independent variables of interest. As an initial test of theoretical predictions, we regress subjects' bids on intrinsic values, which are the equilibrium bids (Table 4, Model 1). The coefficient on intrinsic value is highly significant though less than 1 ($p < .01$). However, contrary to theoretical predictions, the fixed effects are significant in a majority of cases and generally positive.

To understand why subjects deviate from equilibrium predictions, Models 2 and 3 incorporate additional potential variables. First, we include dummy variables for the two types of auctions with externalities: $\xi = 0.5$ (differentiation) and $\xi = 1.0$ (homogeneity). The results (Model 2) suggest that bids are higher relative to equilibrium predictions when externalities are present. Additionally, deviations from equilibrium are similar in both auctions with externalities as the coefficients on the two dummy variables are not highly significantly different from each other ($p = .06$).

Since the equilibrium bid additively incorporates a subject's profit from winning and from losing the auction, we explore how each of these components contributes to a subject's bid. The winning value (Equation 5) is equal to the added profit from winning the auction and decreasing one's costs. In the case of a single license offered for sale, this profit improvement is always positive. The losing value (Equation 6) is the change in profit when another bidder who is presumed to have the same signal wins the auction. This losing value is equal to zero when $\xi = 0$ since rival firms have no impact on profit, and is negative in the other cases. To understand the relative weight subjects place on winning and losing, we regress bids on winning value and losing value separately (Model 3). Since the equilibrium bid is the difference between the winning and losing values, we predict that the coefficients on winning value and losing value are equal to 1 and -1. Subjects appear to

	Censored regression models			
	1	2	3	4
Intrinsic Value	0.843*** (0.024)	0.846*** (0.024)		
Winning Value			0.818*** (0.020)	0.910*** (0.018)
Losing Value			−1.091*** (0.119)	
Lose 50				−1.187*** (0.136)
$\xi = 0.5$ (differentiation)		584*** (149)		
$\xi = 1.0$ (homogeneity)		297*** (150)		
Average fixed effect	679	371	669	104
% of fixed effects significant at < 0.05	55.6%	38.9%	55.6%	27.8%

Note: dependent variable is a subject's bid. Standard errors are shown in parentheses.
*** All coefficients are significant at 0.01. $N = 585$.

Table 4: Estimation of bids in the sale of one license

place a weight less than 1 on the winning value but they place a weight not significantly different from −1 on the losing value ($p = .44$).

Again in Model 3 we note that the fixed effects are significant for a majority of the subjects, implying that bids are not derived solely from the values from winning and losing the auction. We consider another specification that might provide an explanation. While determining the profit from winning an auction for a single license is straightforward – it is the profit given one's signal – determining the expected profit from losing requires assumptions about what signal the winner is likely to have. In equilibrium, the appropriate "assumption" is that the winner has an identical signal. That is, conditional on the information revealed by the fact that I lose the auction, in equilibrium, I bid as if the winner's signal is equal to mine. Yet, we cannot necessarily expect this level of sophistication from bidders. Instead, it is quite possible that subjects formulate bids by assuming that the winner has some signal that is independent of their own. A mean (or, in this context, equivalently, the median) may serve as a reasonable focal point, so that a subject may think "If I lose, I don't know what signal the winner has, but it is likely to be 50 since that's the average signal." We construct a variable, *lose 50*, representing the reduction in profit if the subject loses the auction and the winner has the mean signal (50). This variable takes on only one of three values corresponding to the three possible values of ξ and is equal to 0 in the monopoly condition ($\xi = 0$).

Model 4 shows the results with the simple heuristic *lose 50* instead of the losing value. The coefficient on *lose 50* is not significantly different from -1 ($p = .17$). Additionally, the average of the fixed effects for this specification is quite low relative to other models and insignificant for nearly three out of four subjects. These data suggest that subjects bid *as if* they (almost) properly account for winning value and assume that, if they lose, the winner possesses the mean signal.

Result 3 *In auctions of one license, subjects incorporate the winning value and the losing value somewhat in accord with theoretical predictions. However, the data are also consistent with a model in which subjects assume that a competitor who wins a license has the mean signal.*

The main departure from theoretical predictions comes from insufficient weight being placed on the winning value. While, in all estimations, this parameter is numerically close to one, it is also, in all cases, statistically significantly less than one, leading generally to underbidding in the sale of one license.

5.3 Auctions of Two Licenses

The sale of two licenses presents subjects with the need to estimate not only winners' private information if they lose, but also to predict what the *other* winner's signal might be if they win. We investigate next whether bidder behavior changes in this environment compared to the sale of one license. Our estimation parallels that for the sale of one license (Table 5).

Again we find (in Model 1) that the coefficient on intrinsic value is significant but statistically less than the theoretical prediction of 1 ($p < .01$). However, when compared to the auctions of one license (where the coefficient on intrinsic value is 0.843), there seems to be a stronger co-movement between the bid and the corresponding intrinsic values in the auction for two licenses. Incorporating dummy variables for the auctions with externalities, Model 2 suggests overbidding in the presence of externalities, and this overbidding is greater than in the sale of one license. When considering the components of the intrinsic value separately (Model 3), the coefficient on winning value is again significantly closer to the theoretical prediction of 1 than in the sale of one license. However, the coefficient on losing value also grows in magnitude (in absolute value).

We again consider bidding behavior when subjects substitute the average signal for unknown signals of their competitors (Model 4). This is more complex than in the sale of one license since this not only is incorporated in losing value (assuming both winners each have a signal of 50), but also requires making an assumption about the other winner if the subject wins. The variable *win 50* is a subject's value from winning if the other winner has the average signal. Thus, *win 50* substitutes for winning value when subjects utilize a simple heuristic for determining the profit from winning the auction. Analogous to the sale of one license, we find that this model yields parameters on *lose 50* not significantly different from -1 ($p = .32$), though positive fixed effects persist, and fixed effects are significant for nearly half of our subjects.

	Censored regression models			
	1	2	3	4
Intrinsic Value	0.897***	0.934***		
	(0.024)	(0.024)		
Winning Value			0.886***	
			(0.023)	
Losing Value			−1.336***	
			(0.080)	
Win 50				0.908***
				(0.024)
Lose 50				−1.101***
				(0.100)
$\xi = 0.5$ (differentiation)		1205***		
		(179)		
$\xi = 1.0$ (homogeneity)		923***		
		(180)		
Average fixed effect	1219	328	744	623
% of fixed effects significant at < 0.05	64.1%	46.2%	53.8%	48.7%

Note: dependent variable is a subject's bid. Standard errors are shown in parentheses.
*** All coefficients are significant at 0.01. $N = 585$.

Table 5: Estimation of bids in the sale of two licenses

Result 4 *In auctions of two licenses, subjects appear to incorporate the winning value in a manner close to theoretical predictions while they overemphasize the role of losing value. The data are also consistent with a model in which subjects assume that competitors who win a license have the mean signal.*

5.4 Revenues

Previously, we found evidence of overbidding for auctions of two licenses and underbidding in auctions of one license. This need not imply a similar result for revenue since revenue also depends on the distribution of bids. We now turn to exploring the mean and distribution of observed revenues.

Revenues from the experiment are reported in Table 6. Observed revenues do not differ from predictions in the sale of one license. In the sale of two licenses ($k = 2$), we observe significantly higher revenues than predicted in the presence of externalities. This result is potentially sensitive to process of matching groups of subjects. Observed revenues represent the outcome of a single

	Predicted Revenue	*Observed Revenue*	*Recombinant Revenue*
k=1			
monopoly	5750	5350	5197
($\xi = 0$) $N = 180$		(0.28)	(0.08)
differentiation	5333	5483	5513
($\xi = 0.5$) $N = 180$		(0.64)	(0.50)
homogeneity	6500	6333	6402
($\xi = 1$) $N = 180$		(0.77)	(0.78)
k=2			
monopoly	5500	5298	5499
($\xi = 0$) $N = 195$		(0.51)	(0.98)
differentiation	4519	5657	5685
($\xi = 0.5$) $N = 195$		(0.00)	(0.02)
homogeneity	4250	4965	4690
($\xi = 1$) $N = 195$		(0.01)	(0.32)

In parentheses are p-values for estimated revenues compared to predicted revenues, two sided.

Table 6: Estimated and Predicted Revenues

matching of subjects' bids into groups of three for each auction. An auctioneer interested in expected revenue may wish to know what revenue would occur over many such matchings. For robustness, we also estimate revenues using the recombinant estimator first proposed by Mullin and Reiley (2006), similar to a bootstrap procedure which accounts for the correlations across resamplings due to the same subject appearing in multiple samples. Results with the recombinant estimator cast doubt on the significance of the two license $\xi = 1$ case though confirm that, when $\xi = 0.5$, revenues exceed theoretical predictions. Both observed revenues and the recombinant revenue estimates indicate that revenues are broadly consistent with corresponding predicted revenues; there is, however, weak evidence that experimental revenues exceed predicted revenues for auctions of two licenses in the presence of externalities.[6] This implies that an inventor interested in using theoretical benchmarks to predict revenue or to decide how to structure the auction may wish to adjust predicted revenue upwards for an auction of two licenses.

Result 5 *Obtained revenues are broadly consistent with theoretical predictions. There is weak evidence that the model slightly understates revenue for auctions of two licenses when externalities are present.*

Revenues exceeding theoretical benchmarks need not change auction design guidance unless the departures from equilibrium are systematic and severe enough to change the revenue ranking of different auctions. Given the parameters of our model, the predicted optimal number of licenses to auction is 1 for any value of the externality parameter, ξ. However, this result does not hold

[6] As an additional robustness check, we also considered directly the empirical distribution of bids in each treatment. We determine the distribution of the k^{th} highest order statistic from $N = 3$ draws from the distribution where $k = 2$ or 3. The expected value and standard deviation of this distribution implies results similar to those above.

in our experiment. The recombinant revenue and the empirically observed expected revenue are often higher for an auction of two licenses relative to an auction of one license. This is because the obtained revenues are much higher than predicted revenues for auctions of two licenses relative to auctions of one license. If such an effect holds generally, then it may be optimal for the seller to sell one license only for a subset of the parameter values for which the model predicts that the optimal number of licenses is one.

Result 6 *In the presence of externalities, theoretical predictions understate the relative advantage of selling two licenses.*

5.5 Comparison to a Model of Heuristic Bidding

In the previous section, we conclude that *mean* auction revenues in our experiment correspond quite closely to predictions. In this section, we show that the *distribution* of revenues departs systematically from what we should observe in theory and demonstrate that both this observation and the slight overbidding in the sale of two licenses are consistent with the simple heuristic bidding model.

In Figure 2, we compare the distribution of auction revenues if subjects bid according to theoretical predictions with the simulated distribution of revenues from the experiment.[7] In both monopoly cases, the distributions mostly coincide. We observe several differences in the distributions in the presence of externalities: (i) for auctions of one license, the distribution of observed revenues display an s-shape, lying below predictions for lower revenues and above predictions for higher revenues; (ii) for auctions of two licenses, the distribution function of observed revenues lies below theoretical revenues, except for a small region in the lower end of the distribution. Below, we propose a possible explanation for such differences.

In the model with rational bidders, firm i's bid is the intrinsic value which is the difference between its profit from winning and its profit from losing, when the marginal winner has the same signal as firm i. As suggested by the regression results in the previous sections, it is plausible that instead of bidding the intrinsic value as the model predicts, each bidder adopts a simpler heuristic and assumes that a winner of the auction has the mean signal. Then, each bidder bids

$$\tilde{b}_k\left(\theta_i;\xi\right) = \Pi\left(\theta_i;\theta_{(1)}^{n-1}=50,\ldots,\theta_{(k-1)}^{n-1}=50,\xi\right) - \Pi\left(0;\theta_{(1)}^{n-1}=50,\ldots,\theta_{(k)}^{n-1}=50,\xi\right) \tag{15}$$

which is the difference between her profit from winning and her profit from losing when each winning competitor has the expected value of the signal. In the experiment, the signal of each bidder is independently uniformly distributed between 0 and 100; hence, the expected value of the signal of each bidder is 50. We call $\tilde{b}_k$ of equation (15) the *heuristic value*.

Figure 2 also contains plots of predicted revenues if subjects bid according to their heuristic values. Heuristic bids are equal to equilibrium bids in the monopoly cases, since competitors' signals

[7] The simulated distributions are obtained from 10,000 samples for every treatment, each composed of the bids of three subjects.

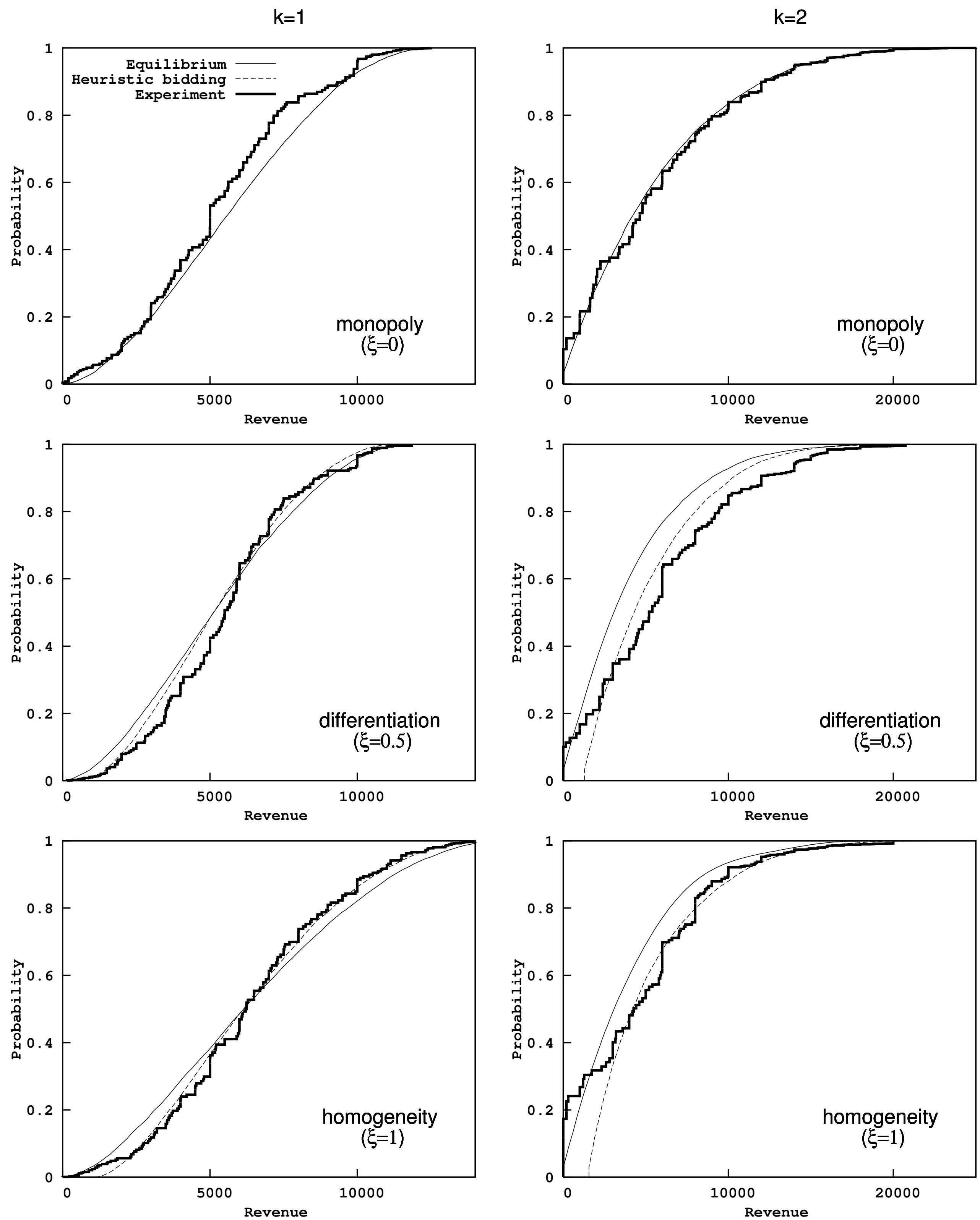

Figure 2: Simulated Distribution of Revenues from equilibrium bids (Intrinsic Values), heuristic bids and observed bids.

	Heuristic Predicted Revenue	*Observed Revenue*	*Recombinant Revenue*
k=1			
monopoly	5750	5350	5197
($\xi = 0$) $N = 180$		(0.28)	(0.08)
differentiation	5340	5483	5513
($\xi = 0.5$) $N = 180$		(0.65)	(0.51)
homogeneity	6531	6333	6402
($\xi = 1$) $N = 180$		(0.56)	(0.70)
k=2			
monopoly	5500	5298	5499
($\xi = 0$) $N = 195$		(0.51)	(0.98)
differentiation	5309	5657	5685
($\xi = 0.5$) $N = 195$		(0.28)	(0.42)
homogeneity	5500	4965	4690
($\xi = 1$) $N = 195$		(0.06)	(0.07)

In parentheses are two-sided p-values for estimated revenues compared to heuristic predicted revenues.

Table 7: Estimated and Predicted Heuristic Revenues

do not enter one's bidding function. In the presence of externalities, heuristic bidding reflects both the s-shape of observed bids in the sale of one license[8] and higher bids (a lower curve) in the case of two licenses.[9]

In Table 7, we present predicted revenues if each bidder bids her heuristic value. Notably, experimental revenues (and their recombinant estimates) are never significantly different from those predicted by heuristic bidding at 5%. In the sale of one license with externalities, heuristic bidding marginally increases predicted revenue (by less than 1%). In the sale of two licenses, this heuristic implies significantly greater bids (17% and 29% for $\xi = 0.5, 1$). These observations are qualitatively consistent with our finding that the theoretical predictions are in line with revenues for the sale of one license but may underestimate revenue in the sale of two licenses.

6 Discussion

A challenge for auctions models is that people, be they experimental subjects or decision-makers in the "real world," rarely exhibit the level of sophistication required for equilibrium calculations. We find evidence that our subjects adopt simple heuristics, acting as if winners' signals are equal to the mean of the distribution of signals. Fortunately, we find that this need not imply significant

[8] For an auction of one license, the profit from winning a license is the same for the intrinsic value and the heuristic value. To calculate the profit from losing, the equilibrium assumption that the winner has the same signal as the bidder is replaced with the heuristic assumption that the winner always has a signal of 50, leading to higher bids for lower signals and lower bids for higher signals than in equilibrium.

[9] For an auction of two licenses, the profit from winning a license is always higher under the heuristic assumption (the other winner has a signal of 50) than in equilibrium (the other winner has a signal higher than mine). The profit from losing is again ambiguous but given the parameters of the model, the intrinsic value is always lower than the heuristic value.

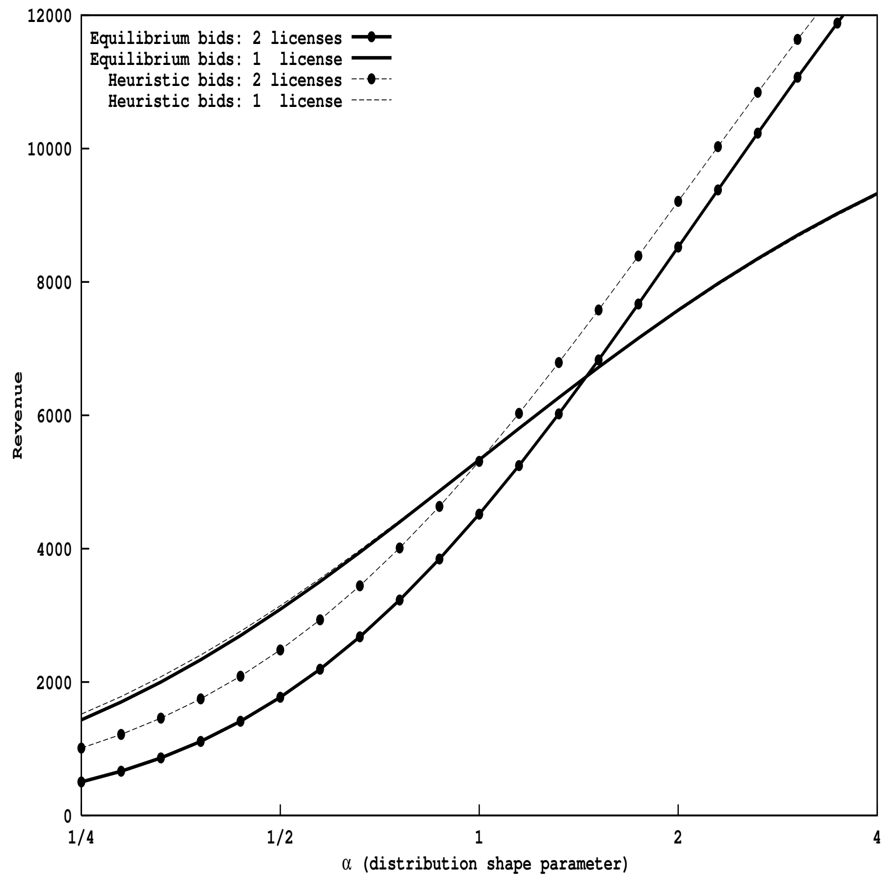

Figure 3: Expected revenues for the sale of one and two licenses under equilibrium and heuristic bidding for the differentiation case.

departures from equilibrium revenues.

In the sale of one license, both equilibrium and heuristic bids imply nearly identical revenues, and these are observed in our experiments. For auctions of two licenses, there is weak evidence that the model slightly underpredicts revenue. The observed overbidding is, however, in line with the heuristic bidding model. How does an auctioneer choose whether to sell one or many licenses if he is unaware of whether participants bid rationally? To examine this question, we replace the assumption in the experiment that the distribution of cost savings is uniform with a special case of the beta distribution; let θ_i be distributed on $[0, 100]$ according to $F(\theta) = (\frac{\theta}{100})^{\alpha}$. When $\alpha = 1$, we recover the uniform distribution. Higher values of alpha imply a greater likelihood of larger cost savings. Again let market demand be given by $p_i = 300 - q_i - \xi \sum_{j \neq i} q_j$ with each firm having a pre-auction marginal cost of $\overline{c} = 100$. Figure 3 compares the revenue predictions when $\xi = 1/2$ under both heuristic and equilibrium bidding. Revenues in the sale of one license are nearly identical under both bidding rules. In the sale of two licenses, heuristic bidders lead to greater revenue.

Of particular import is the point at which selling two licenses dominates the sale of one license. Under the heuristic bidding rule, the sale of two licenses is better whenever $\alpha > 1.02$ and under

Figure 4: Optimal number of licenses to auction under equilibrium and heuristic bidding. In Region I, both models favor the sale of two licenses. In Region II, only the heuristic bidding model favors the sale of two licenses. In Region III, both models favor the sale of one license.

equilibrium bidding, the sale of two licenses is better when $\alpha > 1.45$, confirming our earlier intuition that the sale of two licenses is optimal under a broader sense of parameters when subjects bid according to the simple heuristic. Figure 4 summarizes the predictions of the two bidding models for a range of distributions (α) and externality parameters (ξ). In short, the optimal number of licenses to auction is the same under both models (Region I and Region III) except in a narrow band (Region II) in which the sale of two licenses is preferred under heuristic bidding but one license is optimal under equilibrium bidding.

Our results imply that theory is a useful predictor of revenues in the sale of one license, but a model in which subjects assume that each competing winner has the mean signal can act as a good predictor of auction revenues in multi-license auctions. Because in an auction of two licenses, the heuristic model predicts higher revenues than in equilibrium, it may be optimal to sell one license for a smaller set of parameter values than what we predict theoretically.

Appendix

Proof of Proposition 1

Suppose the firms other than firm i bid according to $b_k(\cdot;\xi)$ and suppose firm i with signal θ_i bids b_i. In an increasing symmetric equilibrium, $b_k(\theta;\xi)$ is increasing in θ, and hence, firm i wins a license if and only if

$$b_i > b_k\left(\theta_{(k)}^{n-1};\xi\right). \tag{16}$$

Notice that the inequality in (16) is equivalent to the following condition:

$$\theta_{(k)}^{n-1} < b_k^{-1}(b_i;\xi).$$

Hence, the payoff of firm i by bidding b_i, is given by

$$\begin{aligned}
&\int_0^{b_k^{-1}(b_i;\xi)}\int_{\theta_{(k)}^{n-1}}^{\overline{c}}\int_{\theta_{(k-1)}^{n-1}}^{\overline{c}}\cdots\int_{\theta_{(2)}^{n-1}}^{\overline{c}}\left[\Pi\left(\theta_i;\theta_{(1)}^{n-1},\ldots,\theta_{(k-1)}^{n-1},\xi\right)-b_k\left(\theta_{(k)}^{n-1}\right)\right]\\
&\times f_{1.k}^{n-1}\left(\theta_{(1)}^{n-1},\ldots,\theta_{(k)}^{n-1}\right)d\theta_{(1)}^{n-1}d\theta_{(2)}^{n-1}\ldots d\theta_{(k)}^{n-1}\\
&+\int_{b_k^{-1}(b_i;\xi)}^{\overline{c}}\int_{\theta_{(k)}^{n-1}}^{\overline{c}}\int_{\theta_{(k-1)}^{n-1}}^{\overline{c}}\cdots\int_{\theta_{(2)}^{n-1}}^{\overline{c}}\Pi\left(0;\theta_{(1)}^{n-1},\ldots,\theta_{(k)}^{n-1},\xi\right)f_{1.k}^{n-1}\left(\theta_{(k)}^{n-1}\right)d\theta_{(1)}^{n-1}d\theta_{(2)}^{n-1}\ldots d\theta_{(k)}^{n-1}.
\end{aligned}$$

where $f_{1.k}^{n-1}\left(\theta_{(1)}^{n-1},\ldots,\theta_{(k)}^{n-1}\right)$ is the joint density function of $\theta_{(1)}^{n-1},\ldots,\theta_{(k)}^{n-1}$. From the first-order condition, we obtain the following:

$$\begin{aligned}
&\int_{b_k^{-1}(b_i;\xi)}^{\overline{c}}\int_{\theta_{(k-1)}^{n-1}}^{\overline{c}}\cdots\int_{\theta_{(2)}^{n-1}}^{\overline{c}}\left[\Pi\left(\theta_i;\theta_{(1)}^{n-1},\ldots,\theta_{(k-1)}^{n-1},\xi\right)-b_k\left(b_k^{-1}(b_i;\xi)\right)-\Pi\left(0;\theta_{(1)}^{n-1},\ldots,b_k^{-1}(b_i;\xi)\right)\right]\\
&\times f_{1.k}^{n-1}\left(\theta_{(1)}^{n-1},\ldots,\theta_{(k-1)}^{n-1},b_k^{-1}(b_i;\xi)\right)d\theta_{(1)}^{n-1}d\theta_{(2)}^{n-1}\ldots d\theta_{(k-1)}^{n-1}\\
=\;&E\left[\Pi\left(\theta_i;\theta_{(1)}^{n-1},\ldots,\theta_{(k-1)}^{n-1},\xi\right)-b_k\left(b_k^{-1}(b_i;\xi)\right)-\Pi\left(0;\theta_{(1)}^{n-1},\ldots,b_k^{-1}(b_i;\xi)\right)|\theta_{(k)}^{n-1}=b_k^{-1}(b_i;\xi)\right]\\
=\;&0.
\end{aligned}\tag{17}$$

In a symmetric equilibrium, firm i has the same bidding strategy as its competitors, and hence,

$$b_i = b_k(\theta_i;\xi). \tag{18}$$

Notice that the condition above is equivalent to the following condition:

$$b_k^{-1}(b_i;\xi)=\theta_i. \tag{19}$$

Substituting (18) and (19) in (17), we obtain that

$$b_k(\theta_i;\xi) = E\left[\Pi\left(\theta_i;\theta^{n-1}_{(1)},\ldots,\theta^{n-1}_{(k-1)},\xi\right) - \Pi\left(0;\theta^{n-1}_{(1)},\ldots,\theta^{n-1}_{(k)},\xi\right)|\theta^{n-1}_{(k)} = \theta_i\right] = V_k(\theta_i,\xi)\,. \quad (20)$$

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