1. Problem 10ex
A manufacturer needs to produce cylindrical cans which contain 1000 cubic units. Find the optimal height and diameter for the cans so that the amount of material used to produce such cans is minimized.

2. Problem 10ex
Find two non-negative numbers $x$ and $y$ such that $2x + 3y = 1$ and $xy$ is maximized.

3. Problem 10ex
Find two non-negative numbers $x$ and $y$ such that $2x + 3y = 1$ and $x^2y$ is maximized.

4. Problem 10ex
Find two non-negative numbers $x$ and $y$ such that $2x + 3y = 1$ and $xy^2$ is maximized.

5. Problem 10ex
Find two non-negative numbers $x$ and $y$ such that $2x + 3y = 1$ and $x^k y^l$ is maximized.

6. Problem 10ex
Let $a > 0$ and $b > 0$. Find two non-negative numbers $x$ and $y$ such that $ax + by = 1$ and $xy$ is maximized.

7. Problem 10ex
Let $a > 0$ and $b > 0$. Find two non-negative numbers $x$ and $y$ such that $ax + by = 1$ and $x^2y$ is maximized.

8. Problem 10ex
Let $a > 0$ and $b > 0$. Find two non-negative numbers $x$ and $y$ such that $ax + by = 1$ and $xy^2$ is maximized.

9. Problem 10ex
Let $a > 0$ and $b > 0$. Find two non-negative numbers $x$ and $y$ such that $ax + by = 1$ and $x^k y^l$ is maximized.

10. Problem 10ex
An oil company is to build a pipeline from the off-shore platform and the nearby oil refinery in the port of city Pip. Design a route for the pipeline so that you minimize the cost if you know the following:
- off-shore platform is 12 miles from the cost.
- the distance between the city and the platform is 22 miles.
- the coastline is straight.
- cost of pipeline under water is 2.5m per mile.
- cost of pipeline on the land is 1.8m per mile.
11. Problem 20ex

The same problem as above with different data
- off-shore platform is \( A \) miles from the coast.
- the distance between the city and the platform is \( B \) miles.
- the coastline is straight.
- cost of pipeline under water is \( c_u \) dollars per mile.
- cost of pipeline on the ground is \( c_l \) per mile.

Further, let \( \lambda = \frac{c_u}{c_l} \). Find for which values of lambda it is more prudent to built the pipeline straight from the platform to the city.

12. Problem 10ex

Find points on the curve \( x^2 - y^2 = 1 \) which are closest to the point \((0,2)\).

13. Problem 20ex

A pipe of negligible diameter is to be carried horizontally around the corner from a hallway 8ft wide into a hallway 4ft wide. What is the maximum length that the pipe can have?

14. Problem 20ex

A scientist is making \( N \) observations, \( x_1, x_2, x_3, \ldots, x_N \). To estimate the data the scientist want to use the least square method. In order to do so he has to find an estimator \( \bar{x} \), which will minimize the following function:

\[
E(x) = (x_1 - x)^2 + (x_2 - x)^2 + (x_3 - x)^2 + (x_4 - x)^2 + \ldots + (x_N - x)^2
\]

Find the estimator \( \bar{x} \).