Sequences

In the following Problems determine whether the following sequences are monotone, eventually monotone, bounded and/or convergent.

1 Problem
\[ a_n = \frac{2n + 3}{3n + 1} \quad n = 1, 2, \ldots \]

2 Problem
\[ a_n = \frac{2n^2 + 3}{3n + 1} \quad n = 1, 2, \ldots \]

3 Problem
\[ a_n = \frac{2n + 3}{3n^2 + 1} \quad n = 1, 2, \ldots \]

4 Problem
\[ a_n = \frac{n^2 + 3}{n^2 - 2n + 1} \quad n = 2, 3, \ldots \]

5 Problem
\[ a_n = \frac{n}{e^n} \quad n = 1, 2, 3, \ldots \]

6 Problem
\[ a_n = \frac{2^n}{2^n + 1} \quad n = 1, 2, 3, \ldots \]

7 Problem
\[ a_n = \frac{19^n}{n!} \quad n = 1, 2, 3, \ldots \]

8 Problem
\[ a_n = \frac{n^n}{n!} \quad n = 1, 2, 3, \ldots \]
9 Problem
\[ a_n = \frac{5^n}{2(n^2)} \quad n = 1, 2, 3, \ldots \]

10 Problem
Let \((a_n)\) be a sequence defined recursively by \(a_1 = 0\) and
\[ a_{n+1} = \sqrt{3 + a_n} \]
Determine whether the sequence is monotone, eventually monotone, bounded and/or convergent and find its limit.

11 Problem
Let \((a_n)\) be a sequence defined recursively by \(a_1 = 9\) and
\[ a_{n+1} = \sqrt{k + a_n} \]
for some \(k \in \mathbb{R}\). Determine for which values of \(k\) the sequence is convergent.

12 Problem
Let \((a_n)\) be a sequence defined recursively by \(a_1 = 1\) and
\[ a_{n+1} = \frac{1}{2} \left( a_n + \frac{3}{a_n} \right) \]
Determine whether the sequence is monotone, eventually monotone, bounded and/or convergent and find its limit.

13 Problem
Let \((a_n)\) be a sequence defined recursively by \(a_1 = 1\) and
\[ a_{n+1} = 1 + \frac{1}{a_n} \]
(a) Show that the sequence is bounded.
(b) Show that the subsequences \((a_{2n})\) and \((a_{2n+1})\) are monotone.
(c) Show that the subsequences \((a_{2n})\) and \((a_{2n+1})\) are convergent and find their limits.
(d) Show that the sequence \((a_n)\) is convergent and find its limit.