Calculus 2 MATH2202
Fall 2009
Final Test - Take-Home Part

Name: ____________________________
Nickname: _________________________

Group A

(10 pts) Prove the Taylor Polynomial Theorem.

**Theorem 1.** Let \( f(x) \in C^{n+1}(I) \) where \( I \) is an interval and let \( a \in I \). Then

\[
f(x) = \sum_{k=0}^{n} \frac{f^{(k)}(a)}{k!} (x - a)^k + \frac{f^{(n+1)}(c)}{(n+1)!} (x - a)^{n+1}
\]

where \( c \in I \).

Write all details of the proof. In the proof you need to use the Rolle’s Theorem. State the Rolle’s Theorem.

Group B

**Problem 1 - 5pts**

Sum the following series

\[
\frac{3^3}{5} - \frac{3^4}{7} + \frac{3^5}{9} - \frac{3^6}{11} + \cdots
\]

Hint: Use the Taylor series for \( \arctan \).

**Problem 2 - 5pts**

Using the power series method to solve the differential equation

\[
y' + xy = 0 \quad y(0) = 1
\]

Write the solution in the form of a power series and then recognize what function it represents.

Group C

(4pts) Find the Taylor series of \( \frac{1}{\sqrt{1 + x}} \).

(3pts) Use the Taylor series from (a) to find the Taylor series for \( g(x) = \frac{x}{\sqrt{1 + x^2}} \).

(3pts) Find \( g^{(16)}(0) \).