MATH2203 MULTIVARIATE CALCULUS

Test 1

Name: ____________________

Nickname: ________________

Instructions

Every problem in the test consists of two parts: **Easier** and **Harder**. **Easier** carries 12 points while **Harder** carries 20 points. You can submit only one of the two for each problem.

1. Problem

Easier:
Let T be a triangle in 3D with vertices at $P = (1, 2, 1)$, $Q = (2, 5, 1)$ and $R = (7, 2, 2)$. Find the area of the triangle.

Harder:
Let S be a sphere in 3D with the center at origin and radius $\sqrt{14}$. Check that the point $(1, 2, 3)$ is on S and find the equation of the plane tangential to S.

2. Problem

Easier:
Let u and v be two vectors in $\mathbb{R}^n$ and $\lambda \in \mathbb{R}$. Prove

$$||u - v|| \leq ||u|| + ||v||.$$

Hint: Use Schwartz-Cauchy- Bunjakovsky inequality

$$|u \cdot v| \leq |u||v| \quad \text{for all } \ u, v \in \mathbb{R}^n$$

Harder:
Let u_j for $j = 1, 2.., n$ are vectors in $\mathbb{R}^N$ for some $N \geq n$. Prove
The General Pythagorean Theorem:
Let vectors u_j be mutually orthogonal, i.e., $u_i \cdot u_j = 0$ for all choices of i and j, and $i \neq j$. Then

$$||u_1 + u_2 + u_3 + \cdots + u_n||^2 = ||u_1||^2 + ||u_2||^2 + ||u_3||^2 + \cdots + ||u_n||^2$$

3. Problem

Easier:
Let a and b be vectors in 3D.Show that

$$(a-b)\times(a+b)=2(a\times b)$$

Harder:
Let a, b, c be three 3D vectors such that $a \times b = a \times c$. Prove that

$$(a-b+c)\times(a+b-c)=0$$

4. Problem

Easier:
Let Z_F be a set of all zeros of a function F. Let $P(x, y) = x^2 - y + 1$. Describe and sketch the Z_P.

Harder:
Let Z_F be a set of all zeros of a function F. Let $P(x,y) = \big(x^2+y^2-1\big)\big((x-5)^2+(y-12)^2-144\big)$. Describe and sketch the Z_P.

5. Problem

Easier:
Let Z_F be a set of all zeros of a function F.Let P be a two variable polynomial. Show that $Z_{P^n} = Z_P$ for any positive integer n.

Harder:
Let Z_F be a set of all zeros of a function F. Let $g(t)$ be univariate function and $P(x, y)$ a polynomial in two variables. Further let $Z_g = \{0\}$. Show that

$$Z_{g(P)} = Z_P.$$

If the condition on Z_g is relaxed so that $Z_g \supset \{0\}$ which one of the two is right $Z_{g(P)} \subset Z_P$ or $Z_{g(P)} \supset Z_P$. Justify your answer.