# MATH2203 MULTIVARIATE CALCULUS

# Final Test 1

Name: ____________________
Nickname: ____________________

## Instructions

Every problem in the test consists of two parts: **Easier** and **Harder**. **Easier** carries 12 points while **Harder** carries 20 points. You can submit only one of the two for each problem.

### 1. Problem

**Easier:**
Let $T$ be a triangle in 3D with vertices at $P = (1, 2, 2)$, $Q = (2, 5, 3)$ and $R = (7, 12, 5)$. Find the area of the triangle.

**Harder:**
Let $S$ be a sphere in 3D with the center at origin and radius $\sqrt{30}$. Check that the point $(1, 2, 5)$ is on $S$ and find the equation of the plane tangential to $S$.

### 2. Problem

**Easier:**
Let $u$ and $v$ be two vectors in $\mathbb{R}^n$ and $\lambda \in \mathbb{R}$. Prove

$$||u - v|| \leq ||u|| + ||v||.$$

**Hint:** Use Schwartz-Cauchy- Bunjakovsky inequality

$$|u \cdot v| \leq |u||v| \quad \text{for all} \;\; u, v \in \mathbb{R}^n$$

**Harder:**
Let $u_j$ for $j = 1, 2.., n$ are vectors in $\mathbb{R}^N$ for some $N \geq n$. Prove
**The General Pythagorean Theorem**:
Let vectors $u_j$ be mutually orthogonal, i.e., $u_i \cdot u_j = 0$ for all choices of $i$ and $j$, and $i \neq j$. Then

$$||u_1 + u_2 + u_3 + \cdots + u_n||^2 = ||u_1||^2 + ||u_2||^2 + ||u_3||^2 + \cdots + ||u_n||^2$$

## 3. Problem

**Easier:**
Let $a$ and $b$ be vectors in 3D.Show that

$$(a-b)\times(a+b)=2(a\times b)$$

**Harder:**
Let $a, b, c$ be three 3D vectors such that $a\times b=a\times c$. Prove that

$$(a-b+c)\times(a+b-c)=0$$

## 4. Problem

**Easier:**
Let $Z_F$ be a set of all zeros of a function F. Let $P(x,y)=x^2-y+2$. Describe and sketch the $Z_P$.

**Harder:**
Let $Z_F$ be a set of all zeros of a function F. Let $P(x,y)=\big(x^2+y^2-1\big)\big((x-3)^2+(y-2)^2-50\big)$. Describe and sketch the $Z_P$.

## 5. Problem

**Easier:**
Let $Z_F$ be a set of all zeros of a function F.Let P be a two variable polynomial. Show that $Z_{P^n}=Z_P$ for any positive integer $n$.

**Harder:**
Let $Z_F$ be a set of all zeros of a function F. Let $g(t)$ be univariate function and $P(x,y)$ a polynomial in two variables. Further let $Z_g=\{0\}$. Show that

$$Z_{g(P)}=Z_P.$$

If the condition on $Z_g$ is relaxed so that $Z_g\supset\{0\}$ which one of the two is right $Z_{g(P)}\subset Z_P$ or $Z_{g(P)}\supset Z_P$. Justify your answer.