Problem 1 - 1ex

Let

\[ A = \begin{bmatrix} 2 & 3 \\ 5 & 4 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 9 & 3 \\ 3 & 1 \end{bmatrix} \]

Solve for \( X \) the matrix equation

\[ A + 2X = B \]

Do it first by adding matrices on the left hand side and then comparing the entries of the resulting matrix and matrix \( B \). Then solve it again using matrix algebra.

Problem 2 - 2ex

(a) Let

\[ A = \begin{bmatrix} 2 & 3 \\ 5 & 4 \end{bmatrix} \]

Compute \( A^T + A \).

(b) Show that for any square matrix \( \frac{A^T + A}{2} \) is a symmetric matrix.

Problem 3 - 2ex

Let \( A \) be a square matrix. For which combinations of scalars \( \alpha \) and \( \beta \) the matrix \( \alpha A^T + \beta A \) is a symmetric matrix.

Problem 4 - 1ex

Let \( A \) and \( B \) are two symmetric matrices. Show that \( A + B \) is symmetric.

Problem 5 - 1ex

Let \( A \) and \( B \) are two symmetric matrices. For which combinations of scalars \( \alpha \) and \( \beta \) the matrix \( \alpha A + \beta B \) is a symmetric matrix.

Problem 6 - 1ex

Let \( A \) be a symmetric matrix. Show the matrix \( \alpha I + A \) is a symmetric matrix.
Problem 7 - 2ex

Let $A$ and $B$ are two square matrices. Let $C = A - B$ is a symmetric matrix. What does that tell you about matrices $A$ and $B$?

Problem 8 - 1ex

A matrix is an upper triangular matrix if all its entries under the diagonal are 0. Let $A$ and $B$ are two upper triangular matrices. For which combinations of scalars $\alpha$ and $\beta$ the matrix $\alpha A + \beta B$ is an upper triangular matrix.