Instructions: This is an in-class part of Test 2. The take-home part will be announced on the course web site no later then Wednesday afternoon. For take-home part students will be split into three groups according to their scores on the in-class part of the test. Students who score 14 or higher on the in-class part of the test will be in group A, students who score 8-13 will be in group B and students who score 7 or less in group C. If there are no students who scored 14 or higher on the test the students with 3 best scores will be in group A. The groups and scores will be listed on the course web page. Deadline to submit take-home part is Monday, April 6, before the regular class.

Problem 1
The matrix $A$ is
\[
\begin{bmatrix}
1 & 2 & 4 & 3 & 8 & 6 \\
1 & 1 & 3 & 2 & 5 & 4 \\
1 & 4 & 6 & 5 & 14 & 10
\end{bmatrix}
\]
(a)(2pts) Find the row space, the null space, the column space of a matrix $A$.
(b)(2pts) What is the rank of $A$?
(c)(2pts) Find the column space of $A^T$.

Problem 2. (5pts)
Let
\[
A = \begin{bmatrix} 7 & -2 \\ 0 & 5 \end{bmatrix}
\]
Find the matrix $S$ and $D$ which diagonalize the matrix $A$, i.e., $A = SDS^{-1}$.

Problem 3. Let
\[
T_0 = 0, \quad T_1 = 1, \quad 3T_{n+1} = 26T_n + 9T_{n-1} \quad \text{for all} \quad n \geq 1
\]
Let
\[
u_n = \begin{bmatrix} T_{n+1} \\ T_n \end{bmatrix}
\]
(a)(1pt) Write the recursive vector formula for $u_n$.
(b)(3pts) Find the closed formula for $T_n$.
(c)(1pt) Estimate $T_{60}$.

Problem 4. (5pts) Let $A$ be a 2x2 matrix with eigenvalues 2 and 7, with corresponding eigenvectors $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ and $\begin{bmatrix} 3 \\ 1 \end{bmatrix}$. Compute $A^3$. 