Introduction into Mathematical Systems - 
Fall 2009 
Takehome Test 2

Name: ________________________________
Nickname: ________________________________

Instructions: Check the grade list for the group assignment. The deadline to submit the take-home part is Wednesday, November 11, before the regular class.

Group A

Problem 1 - 6pts
Let $\approx$ be a relation on a set of all integers defined by

$$a \approx b \text{ if } 7 \mid 2a + 5b.$$ 

Is $\approx$ is an equivalence relation? If it is determine the distinct equivalence classes.

Problem 2 - 6pts
Let $P$ and $Q$ be two equivalence relations on a set $\Omega$. Show that $R = P \cap Q$ is also an equivalence relation on $\Omega$. What are the equivalence classes of $R$?

Group B

Problem 1 - 4pts
Prove that $6|n^3 + 3n^2 + 2n$ for all positive integers $n$.

Problem 2 - 4pts
Consider Necklace problem with 6 pearls, i.e, let $A$ be a set of all necklaces with 6 cyan or heather pearls. Define a relation $\triangleright$ between two necklaces as follows $N \triangleright M$ if we can obtain $M$ from $N$ by rotation.
(a) Check the relation is an equivalence relation.
(b) Find all classes of equivalence.
(c) Find the cardinality of all classes of equivalence. How are those numbers related to the number of pearls?
Problem 3 - 4pts
A relation \( \preceq \) defined on the set \( \mathbb{Z} \) by
\[
  n \preceq k \quad \text{if} \quad n^2 + k^2 \text{ is even.}
\]
Is \( \preceq \) an equivalence relation? If it is determine the distinct equivalence classes.

Group C

Problem 1 - 3pts
Let \( a_1 = 0 \) and \( a_2 = 7 \). Let
\[
a_{n+1} = a_{n-1} + 6n^2 + 2
\]
Find the formula for \( a_n \) and then prove it using math induction.

Problem 2 - 3pts
Prove that
\[
(x + y)^4 - (x - y)^4 = 8xy
\]
providing \( x^2 + y^2 = 1 \).

Problem 3 - 3pts
A relation \( \simeq \) defined on the set \( \mathbb{Z} \) by
\[
  n \simeq k \quad \text{if} \quad n^2 + k^2 \text{ is even.}
\]
Is \( \simeq \) an equivalence relation? If it is determine the distinct equivalence classes.

Problem 4 - 3pts
Let \( \approx \) be an equivalence relation on the set \( \Omega = \{a, b, c, d, e, f, g, h\} \). The equivalence classes are
\[
[a] = \{a, b, c\}, \quad [d] = \{d, e, f, g\}, \quad [h] = \{h\}
\]
Reconstruct \( \approx \), i.e., write down all possible elements of the relation.