Introduction into Mathematical Systems
Math3390
Fall 2009
Assignment No.2

Instructions

The group A must do: Tangles-Part 2, Problem 3, and Problem 6.
The group B must do: Tangles-Part 2, Problem 2, and Problem 5.
The group C must do: Problem 1, Problem 4, Problem 7, and Problem 8.

The problems for group A and B are 4 points each and problems for group C are 3 points each.

Tangles-Part 2.

Here is a brief summary of what we have discovered about tangles:

Amber’s or Steven and Jeff’s Conjecture

There are no reducible tangles.

Recall that a reducible tangle is a tangle whose corresponding rational number \( \frac{m}{n} \) is reducible, i.e., \( \gcd(n, m) > 1 \).

In order to prove Amber-Steven-Jeff Conjecture we need:

Faran’s Lemma

Let \( T \) be a non-reducible tangle. Then tangles obtained by turning or twisting \( T \) are non-reducible as well.

In class we have discussed the proof of Faran’s Lemma. This is a “clean version” of the proof we discussed.

Proof: Once the tangle \( T \) is obtained we have two possibilities, either to turn \( T \) or to twist \( T \). We discuss each case separately.

Let \( R \) be a tangle obtained by turning the tangle \( T \). Then \( R \) corresponds to \( -\frac{m}{n} \).

Since \( \gcd(-m, n) = \gcd(n, m) = 1 \) the tangle \( R \) is non-reducible.

Let \( W \) be a tangle obtained by twisting the tangle \( T \). The rational number corresponding to \( W \) is \( \frac{m+n}{m} \).

Since \( \gcd(m+n, m) = \gcd(n, m) = 1 \) the tangle \( W \) is non-reducible. \( \Box \)

In order to complete the proof we need to show that

\[ \gcd(m, n) = 1 \Rightarrow \gcd(m+n, m) = 1 \quad \forall m, n \in \mathbb{N}. \]

This is your assignment.

Assignment Task 1. Prove

\[ \gcd(m, n) = 1 \Rightarrow \gcd(m+n, m) = 1 \quad \forall m, n \in \mathbb{N}. \]
Assignment Task 2. Use Faran’s Lemma to prove the Amber-Steven-Jeff Conjecture.

Sets

1. **Problem**

How many numbers from 1 to 1,000,000 are not divisible by 2 or 3?

2. **Problem**

How many numbers from 1 to 1,000,000 are not divisible by 2 or 3 or 5?

3. **Problem**

How many numbers from 1 to 1,000,000 are not divisible by 2 or 3 or 5 or 7?

4. **Problem**

In a group of 30 people 12 speak English, 14 Spanish, 13 Chinese. Furthermore 5 of them speak English and Spanish, 7 Spanish and Chinese, and 4 speak English and Chinese. Finally there are 3 people who speak all three languages. How many people do not speak any of the three languages?

5. **Problem**

How many 9 digit integers do exist such that their k-th digit counting from left to right is not k and they do not contain the digit 0.
For example: 214398675 is one of those but 249758631 is not since the fifth digit is 5.

6. **Problem**

In the classroom there are 25 students. 17 of them are cyclists, 13 swimmers and 8 are skiers. There are no students involved in all 3 sports. Every cyclist, every swimmer and every skier is either A or B student. In the class there 6 C or less students. Every A student is can participate in at most one sport.
(a) How many A students are in the class?
(b) How many students do participate in swimming and skiing?
7 Problem

Let $T$ be an equilateral triangle. The sides of $T$ have length $d$. At every vertex of $T$ we draw a circle which center is at the vertex and radius is $d$. Find the area of the intersection of three circles.

8 Problem

Find the exact number of tangles which one can create using at most 10 moves.