Introduction into Mathematical Systems
Math3390
Fall 2009
Homework on Logic and Sets

Instructions
This is Homework for Sets and Logic Section. You do not have to submit these problems. However these problems are eligible for Q&A presentation. Each presented problem is worth 5 extra credit points.

Logic

1 Problem
Let \( P \) and \( Q \) be two statements. Determine whether the following identities are true or false. Use truth tables!

- Identity A \((P \land Q)^\sim = P^\sim \land Q^\sim\)
- Identity B \((P \land Q)^\sim = P^\sim \lor Q^\sim\)
- Identity C \((P \lor Q)^\sim = P^\sim \land Q^\sim\)
- Identity D \((P \lor Q)^\sim = P^\sim \lor Q^\sim\)

2 Problem
Let \( P, Q \) and \( R \) be three statements.
(a) Is \( \land \) commutative, i.e., is \( P \land Q = Q \land P \)?
(b) Is \( \land \) associative, i.e., is \( P \land (Q \land R) = (P \land Q) \land R \)?
(c) Are \( \land \) and \( \lor \) distributive, i.e., is \( P \land (Q \lor R) = (P \land Q) \lor (P \land R) \)?

3 Problem
For a logical statement \( P \) find what are \( P \land P, P \lor P, P \land P^\sim, P \lor P^\sim \)?

4 Problem
Use truth table to show that \((P \Rightarrow Q) \land (Q \Rightarrow R)\) is equivalent to \( P \Rightarrow R \).
5  Problem

Prove the statements using one of the three different methods (direct, contraposition, and contradiction).
(a) If $n^3$ is an odd integer then $n$ is an odd as well.
(b) If $n^2$ is divisible by 3 then $n$ is divisible by 3.
(c) If $n$ is odd then $3n + 5$ is even.

6  Problem

Write in symbols, the converse, the contrapositive, and the negation of the following expressions.
(a) $P \Rightarrow (Q \lor R)$
(b) $P \Rightarrow (Q \land R)$
(c) $(P \lor Q) \Rightarrow R$
(d) $(P \land Q) \Rightarrow R$

7  Problem

Write the negation of the following statements.
(a) $\forall x \in \mathbb{R}, \exists n \in \mathbb{N}, |x|^n > 1$.
(b) $\exists A, \forall c, cA$ is tetracoculos.

Sets

8  Problem

Let $A, B$ and $C$ be three sets. Two statements are given.
(a) Determine which statement is true and which one is false.
(b) Then prove the correct statement and disprove the false statement.

Statement 1.

$$(A \cap B) \setminus C = (A \setminus C) \cap (B \setminus C)$$

Statement 2.

$$(A \cap B) \setminus C = (A \setminus C) \cup (B \setminus C)$$

9  Problem

Prove the set identities using three different methods (direct, contraposition, and contradiction).
(a) $A \setminus B = A \cap B^C$.
(b) $(A \cap B)^c = A^c \cup B^c$
(c) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
(d) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
(e) \( A \times (B \cup C) = (A \times B) \cup (A \times C) \).
(f) \( A \times (B \cap C) = (A \times B) \cap (A \times C) \).

10 Problem

Let \( A = \{ \text{all triangles whose sides } a, b, c \text{ satisfy } a^2 + b^2 = c^2 \} \).
Let \( B = \{ \text{all right angle triangles} \} \).
Prove or disprove that \( A = B \).

11 Problem

Let \( A = \{ \text{all points on the circle with radius } R \text{ and center at origin} \} \).
Let \( B = \{ \text{all points } (x, y) \text{ on the plane such that } x^2 + y^2 = 1 \} \).
Prove or disprove that \( A = B \).

12 Problem

Let \( A = \{ \text{all quadratic polynomials } p(x) \mid p(-1) + p(0) + p(1) = 1 \} \).
Let \( B = \{ \text{all quadratic polynomials } p(x) = ax^2 + bx + c \mid 2a + 3c = 1 \} \).
Prove or disprove that \( A = B \).

13 Problem

Let \( A = \{ \text{all polygons such that sum of all interior angles is } 180^\circ \} \).
Let \( B = \{ \text{all triangles} \} \).
Prove or disprove that \( A = B \).