1 Problem C

Prove or disprove the statement:
*Statement:* If $2|a$ and $3|b$ then $6|ab$.

2 Problem BC

Prove or disprove the statement:
*Statement:* Let $n$ be an integer such that $n^2$ is even. Then $n^2$ is divisible by 4.

3 Problem BC

Prove or disprove the statement:
*Statement:* Let $x > 0$. Then
\[
\frac{x}{x+1} < \frac{x+1}{x+2}
\]

4 Problem BC

Prove or disprove the statement:
*Statement:* Let $x > 0$. Then
\[
\frac{x+1}{x} \geq 2
\]

5 Problem BC

Prove or disprove the statement:
*Statement:* Let $n$ be a natural number. Then $n^2 + n$ is even.

6 Problem ABC

Prove or disprove the statement:
*Statement:* Let $n$ be a natural number greater than 1. Then $n! + 1$ is odd.
7 Problem BC

Prove or disprove the statement:
Statement: Let $a, b$ and $c$ are integers such that $a^2 + b^2 = c^2$. Then at least one of two integers $a$ and $b$ is even.

8 Problem ABC

Everyone knows that $3^2 + 4^2 = 5^2$. Prove that do not exist three consecutive natural numbers such that the cube of the largest is equal to the sum of the cubes of the other two.

9 Problem C

Prove by math induction:
Statement: $13^n - 2^n$ is divisible by 11.

10 Problem BC

Prove by math induction:
Statement: $n^4 - 6n^3 + 23n^2 - 18n$ is divisible by 24.

11 Problem ABC

Prove by math induction that the sum of any three consecutive positive cubes is a multiple of 9. For example, $3^3 + 4^3 + 5^3 = 216 = 9 \cdot 24$

12 Problem ABC

Prove by math induction that for each natural number $n$ larger than 2, $2^{n+1} \leq 3^n$.

13 Problem ABC

Prove by math induction that every natural number $n$ larger than 5 can be written as the sum of natural numbers each of which is a 2 or a 3. For example: $17 = 2+2+2+2+2+2+2+3$.

14 Problem ABC

Prove by math induction:
Let $p, q$ are two integers such that $3|(p^2 + q^2)$. Then $3|p$ and $3|q$.

15 Problem ABC

Prove by math induction:
There are infinitely many primes of the form $4n + 3$. 
16 Problem ABC

Prove or disprove:
There do not exist primes a, b, and c such that \( a^3 + b^3 = c^3 \).

17 Problem ABC

Prove that in the group of 21 people there are at least two people born in the same month.

18 Problem ABC

Prove that in the group of 6 people there are at least three people who know each other or group of three people who do not know each other.