introduction into Mathematical Systems
Math3390
Spring 2010
Relations - Assignment

Group A

1 Problem - 5pts
Let $P$ and $Q$ be two equivalence relations on a set $\Omega$. Show that $R = P \cap Q$ is also an equivalence relation on $\Omega$. What are the equivalence classes of $R$?

2 Problem - 5pts
Two polynomials are related if $p(1) = q(1)$. Is this relation an equivalence relation on the set of all polynomials? If it is, determine the distinct equivalence classes.

3 Problem - 5pts
Let $A = \{a, b, c, d, e, f\}$. Let $S = \{(a, f), (d, c), (f, e)\}$. Find an equivalence relation $R$ on $A$ such that $R$ is the smallest equivalence relation containing $S$. The smallest means that if there is another equivalence relation $W$ containing $S$, then $R \subseteq W$.

4 Problem - 5pts
Let $A = [1, 2]$ and $B = [4, 7]$. Prove that $A \equiv B$, i.e., $A$ is equipotent to $B$. Important: When you find the bijection from $A$ to $B$ you need to prove it is a bijection.

Group B

5 Problem - 5pts
Let $\approx$ be a relation on a set of all integers defined by

\[ a \approx b \text{ if } 7 \mid 2a + 5b. \]

Is $\approx$ is an equivalence relation? If it is, determine the distinct equivalence classes.

6 Problem - 5pts
Let $R$ be an equivalence relation on a set $\Omega$. Let $aRb$, $bRc$, and $cRd$. Is then $aRd$? Explain.
7  Problem - 5pts

Let \( \approx \) be an equivalence relation on the set \( \Omega = \{a, b, c, d, e, f, g, h\} \). The equivalence classes are

\[
[a] = \{a, b, c\}, \quad [d] = \{d, e, f, g\}, \quad [h] = \{h\}
\]

Reconstruct \( \approx \), i.e., write down all possible elements of the relation.

8  Problem - 5pts

Let \( A \) be a set of all numbers divisible by 3. Show that \( A \equiv B \), i.e., \( A \) is equipotent to \( B \).

Important: When you find the bijection from \( A \) to \( \mathbb{Z} \) you need to prove it is a bijection.

Group C

9  Problem - 4pts

A relation \( \asymp \) is defined on \( \mathbb{Z} \) by

\[
n \asymp k \quad \text{if} \quad n^2 + k^2 \text{ is even}.
\]

Is \( \asymp \) is an equivalence relation? If it is determine the distinct equivalence classes.

10 Problem - 4pts

Let \( A = \{1, 2, 3, 4\} \). Define a relation on \( A \) which is reflexive and transitive but not symmetric.

11 Problem - 4pts

Let \( |\) be a relation on a set of all integers defined by

\[
a \mid b \quad \text{if} \quad 5 \mid a^2 + 4b^2.
\]

Is \( |\) is an equivalence relation? If it is determine the distinct equivalence classes.

12 Problem - 4pts

A relation \( \approx \) defined on the set \( \mathbb{N} \). We say \( n \approx k \) if both integers contain same power of 2 as a factor in their prime factorization.

For example \( 24 \approx 40 \) because \( 24 = 2^3 \cdot 3 \) and \( 40 = 2^3 \cdot 5 \).

Is \( \approx \) is an equivalence relation? If it is determine the distinct equivalence classes.
Problem - 4pts

Let \( f : \mathbb{N} \rightarrow \mathbb{N} \) be defined as \( f(n) = n^2 + n - 1 \).

(a) Is \( f \) a one-to-one function? Prove or disprove.
(b) Is \( f \) an onto function? Prove or disprove.
(c) Is \( f \) a bijection? Prove or disprove.