Introduction into Mathematical Systems - Spring 2010
Mid-term Takehome Test

Name: ____________________________
Nickname: ____________________________

Instructions: Check the grade list for the group assignment. The deadline to submit the take-home part is Wednesday, March 15, before the regular class.

Group A

Let $X$ and $Y$ be two sets. Let $f : X \rightarrow Y$. For any $A \subseteq X$ we define the image of $A$ under the function $f(x)$, denoted by $f(A)$, as follows:

$$f(A) = \{ y \in Y \mid \exists x \in A \text{ s.t. } y = f(x) \}.$$

Furthermore for any $B \subseteq Y$ we define the preimage of $B$ under the function $f(x)$, denoted by $f^{-1}(B)$, as follows

$$f^{-1}(B) = \{ x \in X \mid f(x) \in B \}.$$

Problem 1 - 2pts

Let $f(x) = x^2 + 1$.
(a) Find $f([0, 1])$.
(b) Find $f^{-1}([0, 1])$.

Problem 2 - 5pts

Prove or disprove:
(a) Let $A$ and $B$ be two subsets of $X$. Then

$$f(A \cap B) \subseteq f(A) \cap f(B)$$

(b) Let $A$ and $B$ be two subsets of $X$. Then

$$f(A) \cap f(B) \subseteq f(A \cap B)$$
Problem 3 - 5pts

Prove or disprove:
Let $A$ be a subset of $X$. Then
$$f^{-1}(f(A)) = A$$

Group B

Problem 1 - 4pts

Let $S = \{ y = ax^2 + bx + c \mid \text{the graph of } y(x) \text{ passes through the points } (1,2) \text{ and } (2,4) \}$
and let $T = \{ y = ax^2 + bx + c \mid 3a + b = a + b + c = 2 \}$. Prove or disprove $S = T$.

Problem 2 - 4pts

Prove or disprove.
The sum of angles in a quadrilateral is $360^\circ$.

Problem 3 - 4pts

Prove or disprove.
Let $x$ and $y$ be two integers. If $3|x$ then $3|[(x + y)^2 - (x - y)^2]$.

Group C

Problem 1 - 3pts

Prove or disprove:
Let $A, B, C$ be sets. Then
$$A \cap (B \setminus C) = (A \cap B) \setminus (A \cap C).$$

Problem 2 - 3pts

Prove or disprove.
Let $x$ and $y$ be two integers. If $5|3x - y$ then $5|y + 2x$.

Problem 3 - 3pts

Prove or disprove.
There is no quadrilateral with four acute angles.
Problem 4 - 3pts

Prove or disprove the logical identity.

\[(P \land Q) \Rightarrow R \quad = \quad (Q \land R^\sim) \Rightarrow P^\sim\]