Introduction into Mathematical Systems - 
Summer 2009 
Test 1 

Name: ________________________________
Nickname: ________________________________

Instructions: This is an in-class part of Test 1. The take-home part will be announced on the course web site no later then Friday afternoon. For take-home part students will be split into three groups according to their scores on the in-class part of the test. Students who score 15 or higher on the in-class part of the test will be in group A, students who score 8-14 will be in group B and students who score 7 or less in group C. If there are no students who scored 15 or higher on the test the students with 3 best scores will be in group A. The groups and scores will be listed on the course web page. Deadline to submit take-home part is Wednesday, July 8, before the regular class.

Problem 1 (4pts) Let $A, B$ be two sets. Prove 
$$(A \setminus B)^c = A^c \cup B$$

Problem 2. (3pts) Using the truth tables prove or disprove the logical identity 
$$(P \land Q^\sim) = P^\sim \lor Q$$

Problem 3. (4pts) Prove the statement:
Let $p, q, r$ be three real numbers such that $p + q + r = 1$. Then 
$$p^3 + 3p^2q + 3p^2r = 3p^2 - 2p^3$$

Problem 4. (4pts) Prove the following statement:
Let $m$ be an integer such that $5|m^3$. Then $5|m$.

Problem 5. (3pts) A coach of the basketball team can choose from 792 different starting teams (5 players). How many players altogether are on the team?

Problem 6. (3pts) The statement is:
There are infinitely many prime numbers.
A proof of the statement is given below. Determine which method of proving has been used in the proof.
Proof. Assume there only finitely many prime numbers. Denote them by $p_1, p_2, \cdots, p_n$. 

Construct a new number $N = p_1 p_2 \cdots p_n + 1$. This number is not divisible by any of the primes in the list. Hence it is a prime number which is not in the list. The theorem is therefore proven.

**Problem 7.** (5pts) A triple of positive integers $(a, b, c)$ is a Pythagorean triple if $a^2 + b^2 = c^2$. Prove or disprove:
If $(a, b, c)$ is a Pythagorean triple then $ab$ is even.