Introduction into Mathematical Systems  
Math3390  
Summer 2010  
Logic - Assignment

Group A

1 Problem - 5pts

Prove or disprove the statement:
Statement:

If $3|m^2$ then $3|m$.

2 Problem - 5pts

Prove that all 16 logical operands can be expressed only in terms of NAND operand.
Hint: Notice that it is not necessary to explicitly write every operand in terms of NAND. It suffices to show it is possible. For example, during the last classroom discussion Josh noticed that all operands can be split in two groups in such way that operands in one group can be obtained by negating operands of the other group. Therefore it suffices to show that one of the group is expressible only in terms of NAND and then used the negation trick we discussed in class to complete the proof.

3 Problem - 5pts

The logical operand NOR is defined as

$$A \text{ NOR } B = (A \lor B)^\sim$$

Express logical expression $A \Rightarrow B$ only in terms of NOR.

Group B

4 Problem - 5pts

Prove or disprove the statement:

If $((A \Rightarrow B) \land (B \Rightarrow C))$ is true then $A \Rightarrow C$ is true.
5 Problem - 5pts
Prove or disprove the statement:

The product of two odd numbers is odd.

6 Problem - 5pts
Write the contrapositive of the statement:

If 3|m and 2|m then 6|m.

Recall that p|m means that m is divisible by p.

Group C

7 Problem - 5pts
Prove or disprove the logical statement:

\[ P \land (Q \lor R) = (P \land Q) \lor (P \land R) \]

8 Problem - 5pts
Prove or disprove the statement:

The sum of two odd numbers is odd.

9 Problem - 5pts
Write the contrapositive of the statement:

If p is a prime then 3 does not divide p.