SECCI\( \text{H} \) DISC RELATIONSHIPS\(^1\)

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ABSTRACT: The problem of estimating bulk light extinction coefficients in surface water impoundments is examined. It is shown that the bulk extinction coefficient can be accurately estimated from the Secchi disc depth. In addition, the equation derived is significantly different from that derived by Poole and Atkins in 1929. This empirical expression for the extinction coefficient is then used to develop a semi-empirical expression for the euphotic depth as a function of the Secchi disc depth.

(KEY TERMS: secchi disc; photic depth; reservoirs.)

INTRODUCTION

The arrival of solar radiation at the surface of a reservoir or lake is an initial point in heat budget computations and subsequent water quality calculations. The solar radiation penetrating to a depth \( z \) below the surface of an impoundment is assumed; e.g., Cole (1975), to be given by:

\[
I_z = I_0 e^{-\eta z}
\]  

where \( I_z \) = solar radiation intensity at depth \( z \), \( I_0 \) = net solar radiation penetrating the surface of the impoundment, and \( \eta \) = bulk extinction coefficient. A second parameter of importance in water quality computations is the euphotic or photic depth, \( z_{EP} \), which is by definition, Cole (1975, p. 117), the depth at which 99 percent of the surface radiation has been dissipated.

Although both \( \eta \) and \( z_{EP} \) can be determined accurately by using a submarine photometer, this method may yield data which are not cost effective in terms of the intended use; e.g., in constructing a heat budget for a reservoir. In the material which follows, it is demonstrated that reasonably accurate values of \( \eta \) and \( z_{EP} \) can be estimated from regression relationships based on the Secchi disc depth in artificial impoundments. It is noted that this work is an extension of the work of Williams, et al. (1980).

EXTINCTION COEFFICIENT

According to Cole (1975) two British oceanographers, Poole and Atkins, derived in 1929 an empirical relationship between the extinction coefficient and the Secchi disk depth; i.e.,

\[
\eta = \frac{1.7}{z_{SD}}
\]  

where \( \eta \) = extinction coefficient \((1/m)\) and \( z_{SD} \) = Secchi disk depth \((m)\). This relationship is still in general use; e.g., Octavio, et al. (1977), even though it was originally derived for ocean waters. With regard to extinction coefficient-Secchi disc relationships, the following comments should be noted. First, Secchi disc readings are influenced by the absorption characteristics of the water and by the dissolved and suspended particulate matter within the water column. Second, although the Secchi disc depth is reasonably independent of the surface light intensity, measurements should be made by midday since the results are usually erratic near dawn and dusk, Wetzel (1975). Thus, since artificial impoundments tend toward greater turbidity due to silt and clay, their optical properties, and hence the Secchi disk depth, tend to differ from natural impoundments. Then based on the foregoing material, it is hypothesized that:

\[
\eta = \frac{c}{z_{SD}}
\]  

where \( c \) = an undetermined or free coefficient. Taking logarithms of both sides of Equation (3) yields:

\[
Y = -x + \phi
\]  

where \( Y = \ln(\eta) \), \( x = \ln(z_{SD}) \), and \( \phi = \ln(c) \). Equation (4) has a single free coefficient, \( \phi \), because the slope is defined as a result of the form of the hypothesis. An additional free coefficient can be added to the model by hypothesizing that:

\[
\eta = \frac{c'}{z_{SD}^\beta}
\]

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\(^1\)Report No. 81069 of the Water Resources Bulletin. Discussions are open until October 1, 1982.

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\[ y = \phi' x + \phi' 
\]

where \( \phi' \) and \( \phi' \) are undetermined coefficients and \( \phi' = \ln(c') \). Equations (4) and (6) are linear, and therefore, the principle of least squares can be used to estimate values of \( \phi, \phi' \), and \( \beta' \).

The data base used to estimate these coefficients consisted of 66 data pairs (\( z_{SD} \) and \( \eta \)) from Lahontan Reservoir near Fallon, Nevada (French, 1981; Cooper, et al., 1981), and 131 data pairs from 11 reservoirs in the Midwest and Southeast (D. T. Williams, U.S. Army Corps of Engineers, Vicksburg, Mississippi, personal communication, June 9, 1980). The results of this analysis are summarized in Table 1 and Figure 1.

**TABLE 1. Summary of Regression Results for Extinction Coefficient.**

<table>
<thead>
<tr>
<th>Assumed Relationship</th>
<th>Coefficient Values</th>
<th>Standard Error of Estimate (transformed variables)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \eta = \frac{c'}{\phi'} ) (Equation 3)</td>
<td>( c' = 1.16 )</td>
<td>0.809</td>
</tr>
<tr>
<td>( \eta = \frac{\beta'}{\phi'} ) (Equation 5)</td>
<td>( c' = 1.21 )</td>
<td>0.250</td>
</tr>
<tr>
<td>( \beta' )</td>
<td>( \beta' = 0.80 )</td>
<td></td>
</tr>
</tbody>
</table>

Figure 1. Extinction Coefficient as a Function of Secchi Disc Depth.

**PHOTIC DEPTH**

If the extinction coefficient can be estimated from the Secchi disc depth, then the photic depth can also be estimated from the Secchi disc depth. Combining Equations (1) and (5) and applying the definition of photic depth yields:

\[ z_E = \frac{4.60}{c'} z_{SD} \]

or

\[ z_E = \Gamma z_{SD} \]

where \( z_E \) is photic depth (m) and \( \Gamma \) = an undetermined coefficient. An alternative formulation for the photic depth could be derived by combining Equations (1) and (5); i.e.,

\[ z_E = \frac{4.60}{c'} z_{SD} \theta' \]

or

\[ z_E = \phi z_{SD} \]

where \( \theta \) and \( \phi \) are undetermined coefficients. Equation (8) is linear and Equation (10) can be transformed to a linear form; therefore, the principle of least squares can be used to estimate values of \( \Gamma, \phi, \) and \( \theta \).

The data base used to determine the coefficients in Equations (8) and (1) consisted of 66 data pairs, \( (z_E \) and \( z_{SD} \) from Lahontan Reservoir, Nevada (French, 1981), 131 data pairs from 11 reservoirs in the Midwest and Southeast (D. T. Williams, U.S. Army Corps of Engineers, Vicksburg, Mississippi, personal communication, June 9, 1980), and 11 data pairs from Topaz Lake, Nevada (J. Cooper, Bioresources Center, Desert Research Institute, Reno, Nevada, personal communication). In view of the caveats previously noted regarding Secchi disc measurements, it should be noted that Topaz Lake is an artificial impoundment. The results of this analysis are summarized in Table 2 and Figure 2.

**TABLE 2. Summary of Regression Results for Photic Depth.**

<table>
<thead>
<tr>
<th>Assumed Relationship</th>
<th>Coefficient Values</th>
<th>Standard Error of Estimate</th>
</tr>
</thead>
<tbody>
<tr>
<td>( z_E = \Gamma z_{SD} ) (Equation 8)</td>
<td>( \Gamma = 2.81 )</td>
<td>0.32</td>
</tr>
<tr>
<td>( z_E = \phi z_{SD} ) (Equation 10)</td>
<td>( \phi = 3.19 )</td>
<td>0.83</td>
</tr>
</tbody>
</table>

**DISCUSSION**

With regard to the relationships developed for estimating the extinction coefficient from Secchi disc depths, the following should be noted. First, a test in the transformed coordinate system demonstrates that the value of the least squares coefficient in Equation (3) is significantly different from the
value given by Cole (1975); i.e., ln (1.16) = 0.148 is significantly different from ln (1.7) = 0.531. Second, a t test in the transformed coordinate system demonstrates that the estimated value of \( \beta \) is significantly different from \(-1\), which is the value given by Cole (1975). Third, since this is a purely empirical analysis, there is no justification for not using the best regression relationship available. If the standard error of estimate in the transformed coordinate system is accepted as a measure of accuracy, then Equation (6) should be used to estimate the extinction coefficient.

![Diagram](image)

**Figure 2. Euphotic Depth as a Function of Secchi Disc Depth.**

With regard to the relationships developed for estimating the photic depth, the following comments are pertinent. First, Equations (8) and (10) are based on empiricism; and therefore, the best relationships should be adapted for use. An F test of the standard errors of estimates for the equations indicates that these parameters are equal. Thus, it is concluded that Equations (8) and (10) are equally valid at this level of testing. Second, the Topaz Lake data appears to be significantly different in comparison with the other data, Figure 2. If Topaz Lake were a natural impoundment, then this difference could be explained in terms of the different optical properties of lake and reservoir water; however, this is not the case. Therefore, at this time, it can only be concluded that the accuracy of the regression relationships decreases with increasing Secchi disc depth. Thus, with regard to photic depth estimation, it is recommended that Equation (8) be used, but with extreme caution when the Secchi disc depth exceeds approximately 1.5 meters.

**CONCLUSIONS**

The material presented here is not intended to replace the sophisticated light penetration equations derived in the past two decades; e.g., Gaume (1975); rather it is intended to provide empirical relationships which may, in some cases, provide estimates of sufficient accuracy for the extinction coefficient and photic depth. The relationships derived provide reasonably accurate fits of the data collected by a number of investigators under diverse conditions in a number of artificial impoundments. It is recommended that these results only be applied in artificial impoundments and that the potential user be aware of their inherent limitations. It is hoped that this information will prove useful to other investigators.

**ACKNOWLEDGMENTS**

Financial support for this research was provided by the Office of Water Research and Technology in two grants under Contract 14-34-0001-0271 and by the United States Geological Survey and the State of Nevada under a co-operative agreement. The authors wish to thank Mr. Dave Williams of the United States Army Corps of Engineers for providing data on reservoirs outside of Nevada.

**LITERATURE CITED**


