Minimum-sized Positive Influential Node Set Selection for Social Networks: Considering Both Positive and Negative Influences

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OUTLINE

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- Greedy Algorithm
- Performance Evaluation
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Motivation

- Example & Applications
- Related Work
- Contributions
**INTRODUCTION**

- What is a social network?
  - The graph of relationships and interactions within a group of individuals
Social network and spread of influence

Social network plays a fundamental role as a medium for the spread of influence among its members

- Opinions, ideas, information, innovation...

Direct Marketing takes the “word-of-mouth” effects to significantly increase profits (facebook, twitter, myspace, ...)
Motivation

MOTIVATION

- **900 million** users, Apr. 2012
- The 3rd largest — “Country” in the world
- More visitors than Google
- Action: Update statues, Create event

- More than **4 billion** images
- Action: Add tags, Add favorites

Social networks already become a **bridge to connect** our daily life and the virtual web space

- 2010, **4 billion** tweets per quarter
- Action: Post tweets, Retweet
Who are the positively influential leaders in a community?

Find minimum-sized node (user) set in a social network that could positively influence every node in the network.
APPLICATIONS

- Smoking intervention program
- Promote new products
- Advertising
- Social recommendation
- Expert finding
- …
**RELATED WORK**

- **Influence Maximization (IM) Problem** [Kempe03]
  - Select $k$ nodes, maximize the expected number of influenced individuals

- **Positive Influence Dominating Set (PIDS)** [Wang11]
  - Minimum-sized nominating set $D$, every other node has at least half of its neighbors in $D$
OUR CONTRIBUTIONS

 Consider both positive and negative influences

 New optimization problem - Minimum-sized Positive Influential Node Set (MPINS)
  o Minimum-sized node set that could positively influence every node in the network no less than a threshold $\theta$

 Propose a greedy algorithm to solve MPINS

 Conduct simulations to validate the proposed algorithm
Problem Definition

- Network Model
- Diffusion Model
- Problem Definition
A social network is represented as an undirected graph. Social influence is represented by the weights on the edges. Nodes start either active or inactive. An active node may trigger the activation of neighboring nodes based on a pre-defined threshold $\theta$. Monotonicity assumption: active nodes never deactivate.
Problem Definition

**DIFFUSION MODEL**

- **Positive influence**
  
  \[ p_{ui}(\mathcal{A}^I(u_i)) = 1 - \prod_{u_j \in \mathcal{A}^I(u_i)} (1 - p_{ij}) \]

- **Negative influence**
  
  \[ p_{ui}(\mathcal{N}^I(u_i)) = 1 - \prod_{u_j \in \mathcal{N}^I(u_i)} (1 - p_{ij}) \]

- **Ultimate influence**
  
  \[ \varphi^I(u_i) = p_{ui}(\mathcal{A}^I(u_i)) - p_{ui}(\mathcal{N}^I(u_i)) \]

\[ \theta = 0.8 \quad \text{Active User} \]
**MINIMUM-SIZED POSITIVE INFLUENCE NODE SET SELECTION PROBLEM (MPINS)**

- **Given**
  - a social network $G(V, E, P(E))$
  - a threshold $\theta$

- **Goal**
  - The initially selected active node set denoted by $I$ could positively influence every other node in the network
  
  $\forall u_i \in V \setminus I, \quad \phi^I(u_i) = p_{u_i}(A^I(u_i)) - p_{u_i}(N^I(u_i)) \geq \theta$

- **Objective**
  - Minimize the size of $I$
Greedy Algorithm

- Contribution Function
- Example
- Correctness Proof
CONTRIBUTION FUNCTION

\[ f(\mathcal{I}) = \sum_{i=1}^{\lvert \mathcal{V} \rvert} \max[\min(g^I(v_i), \theta), 0] \]
TWO-PHASE ALGORITHM

- Maximal Independent Set ($\mathcal{M}$)
- Greedy algorithm

**Algorithm 1 MPINS-GREEDY Algorithm**

**Require:** Social network $G(V, E, P(E))$; a pre-defined threshold $\theta$.

1. Initialize $\mathcal{I} = \mathcal{M}$
2. while $f(\mathcal{I}) < |V|\theta$ do
3. choose $u \in V \setminus \mathcal{I}$ to maximize $f(\mathcal{I} \cup \{u\})$
4. $\mathcal{I} = \mathcal{I} \cup \{u\}$
5. end while
6. return $\mathcal{I}$
Greedy algorithm

**EXAMPLE**

BFS Ordering: \{u_1, u_2, u_3, u_4, u_5, u_6, u_7\}

MIS: \{u_1, u_6\}

\[\theta = 0.8\]
CORRECTNESS PROOF

Theorem 1. Algorithm 1 produces a feasible solution for the MPINS selection problem. To be specific,

1) Algorithm 1 terminates for sure.
2) \( f(\mathcal{I}) = |\mathcal{V}| \theta \) if and only if \( \mathcal{I} \) is a positive influential node set, such that every node (i.e., \( \forall u_i \in \mathcal{V} \)) is positively influenced by nodes in \( \mathcal{I} \) no less than \( \theta \).
Performance Evaluation

- Simulation Settings
- Simulation Results
SIMULATION SETTINGS

- Generate random graph
- Randomly generate the weighs on the edges
- For each specific setting, generate 100 instances of the graph. The results are the average values of these 100 instances
SIMULATION RESULTS – SMALL SCALE

Performance Evaluation
**Simulation Results – Large Scale**

Performance Evaluation
SIMULATION RESULTS

Performance Evaluation
CONCLUSION

- We study MPINS selection problem considering both positive and negative influences, which has useful commercial applications in social networks
- We propose a greedy algorithm to solve the problem
- We validate the proposed algorithm through simulations on random graphs representing small and large size networks
Related References


Q & A