MATH 2202 - Practice Questions for Test 1

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Abstract

This is a list of practice problems for test 1. You should try to answer the questions without looking at your notes. This document is not an indication of how the test will be (in length, type of questions, difficulty, number of questions, ...). It is simply designed to test your knowledge of the material you should know for the test.

1. What is the geometric interpretation of \( \int_{a}^{b} f(x) \, dx \) if \( f \) is a continuous function. Illustrate your explanation with a diagram.

2. Explain in words what you have to do to evaluate \( \int_{a}^{b} f(x) \, dx \) if \( f \) is a continuous function. Illustrate your explanation with an example.

3. If \( v(t) \) represents the velocity of an object (\( t \) expressed in seconds), explain what \( \int_{0}^{10} v(t) \, dt \) represents.

4. Express each integral below in terms of \( f \).

   (a) \( \int f'(x) \, dx \)
(b) \( \int_{a}^{b} f'(x) \, dx \)

(c) \( \frac{d}{dx} \left( \int_{a}^{x} f(t) \, dt \right) \)

5. Answer True or False. If you answer True, explain why. If you answer False, either explain why, or give an example.

(a) If \( f \) and \( g \) are continuous, then
\[
\int_{a}^{b} (f(x) + g(x)) \, dx = \int_{a}^{b} f(x) \, dx + \int_{a}^{b} g(x) \, dx
\]

(b) If \( f \) and \( g \) are continuous, then
\[
\int_{a}^{b} (f(x)g(x)) \, dx = \left( \int_{a}^{b} f(x) \, dx \right) \left( \int_{a}^{b} g(x) \, dx \right)
\]

(c) If \( f' \) is continuous, then
\[
\int f'(x) \, dx = f(x) + C
\]

(d) \( \int_{-2}^{2} e^{x^2} \, dx = 0 \)

6. Evaluate \( \frac{d}{dx} \left( \int_{0}^{x} e^{\tan^{-1} t} \, dt \right) \)

7. Evaluate \( \int_{0}^{1} \frac{d}{dt} \left( e^{\tan^{-1} t} \right) \, dt \)

8. Evaluate each integral below using any method you want.

(a) \( \int_{0}^{2} \frac{dx}{4 + x^2} \)
9. Evaluate (or prove they diverge) the improper integrals below.

(a) \( \int_0^{\infty} \frac{1}{(x + 2)^4} \, dx \)

(b) \( \int_0^{\infty} e^{-2x} \, dx \)

(c) \( \int_{-1}^{1} \frac{1}{2x + 1} \, dx \)

10. Find the derivative of the functions below:

(a) \( F(x) = \int_1^x \sqrt{1 + t^4} \, dt \)
(b) \( G(x) = \int_{x}^{1} \sqrt{1+t^4} \, dt \)

11. Using the tables at the end of your book, evaluate the integrals below:

(a) \( \int \sin^4 x \, dx \)

(b) \( \int \frac{x^4}{\sqrt{x^{10} - 1}} \, dx \)

(c) \( \int \sqrt{e^{2x} - 1} \, dx \)

12. If \( f \) is continuous for all reals, prove that

\[
\int_{a}^{b} f(-x) \, dx = \int_{-b}^{-a} f(x) \, dx
\]