Math 1113 Notes - Functions Revisited

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Abstract
Solutions to the homework on one-to-one and inverse functions

1 Problems

1. Given \( f(x) = \frac{x + 1}{x^2 - 4} \) and \( g(x) = \frac{2}{x} \), find \( f + g \), \( f - g \), \( fg \), \( \frac{f}{g} \), and state their domains.

- \((f + g)(x) = \frac{x + 1}{x^2 - 4} + \frac{2}{x} = \frac{3x^2 + x - 8}{x(x^2 - 4)} \). The domain is \( \mathbb{R} - \{ -2, 0, 2 \} \).
- \((f - g)(x) = \frac{x + 1}{x^2 - 4} - \frac{2}{x} = \frac{-x^2 + x + 8}{x(x^2 - 4)} \). The domain is \( \mathbb{R} - \{ -2, 0, 2 \} \).
- \((fg)(x) = \frac{x + 1}{x^2 - 4} \cdot \frac{2}{x} = \frac{2x + 2}{x(x^2 - 4)} \). The domain is \( \mathbb{R} - \{ -2, 0, 2 \} \).
- \( \left( \frac{f}{g} \right)(x) = \frac{x + 1}{\frac{x^2 - 4}{2x}} = \frac{x + 1}{\frac{x^2 - 4}{2}} = \frac{x^2 + x}{2(x^2 - 4)} \). The domain is \( \mathbb{R} - \{ -2, 2 \} \).

2. Given \( f(x) = x^4 \) and \( g(x) = 3x + 5 \), find \( f \circ g \), \( g \circ f \) and state their domains.

- \((f \circ g)(x) = f(g(x)) = (3x + 5)^4 \). The domain is \( \mathbb{R} \).
- \((g \circ f)(x) = g(f(x)) = 3x^4 + 5 \). The domain is \( \mathbb{R} \).

3. Determine whether the functions below are one-to-one

(a) \( f(x) = 7x - 3 \)
   - one-to-one, passes horizontal line test.

(b) \( g(x) = x^4 - 5 \)
   - Not one-to-one, polynomial of even degree.
(c) \( h(x) = x^2 - 2 \)
Not one-to-one, polynomial of even degree.

4. Assuming \( f \) is a one-to-one function, answer the questions below

(a) Find \( f^{-1}(7) \) if \( f(2) = 7 \)
\( f^{-1}(7) = 2 \)

(b) Find \( f^{-1}(8) \) if \( f(-5) = 8 \)
\( f^{-1}(8) = -5 \)

5. Show that the functions below are inverses

(a) \( f(x) = 2x - 5, \ g(x) = \frac{x + 5}{2} \)
We need to check that \( f(g(x)) = x \) and \( g(f(x)) = x \)

(b) \( f(x) = x^2 - 4 \) and \( x \geq 0, \ g(x) = \sqrt{x+4} \) and \( x \geq -4 \)
We need to check that \( f(g(x)) = x \) and \( g(f(x)) = x \)

6. Find the inverse of \( f \) for each case below.

(a) \( f(x) = 2x + 1 \)
\( f^{-1}(x) = \frac{x - 1}{2} \)

(b) \( f(x) = 4x + 7 \)
\( f^{-1}(x) = \frac{x - 7}{4} \)

(c) \( f(x) = \frac{x - 2}{x + 2} \)
\( f^{-1}(x) = \frac{2x + 2}{1 - x} \)

(d) \( f(x) = \frac{1 + 3x}{5 - 2x} \)
\( f^{-1}(x) = \frac{5x - 1}{3 + 2x} \)

(e) \( f(x) = \sqrt{2 + 5x} \)
\( f^{-1}(x) = \frac{x^2 - 2}{5} \)

(f) \( f(x) = 4 - x^2, \ x \geq 0 \)
\( f^{-1}(x) = \sqrt{4 - x} \)