1.6 Quadric Surfaces

1.6.1 Brief review of Conic Sections

You may need to review conic sections for this to make more sense. You should know what a parabola, hyperbola and ellipse are. We remind you of their equation here. If you have never seen these equations before, consult a calculus book to study them in more detail.

- Parabola: \( y = ax^2 \) or \( x = ay^2 \). The parabola is along the axis corresponding to the variable not being squared. Figures 1.18 and 1.19 illustrate this.

- Ellipse: \( \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \). An ellipse looks like an elongated circle. It extends
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Figure 1.20: Ellipse \( \frac{x^2}{4} + \frac{y^2}{9} = 1 \)

in the x-direction from \(-a\) to \(a\) and in the y-direction from \(-b\) to \(b\). Figure 1.20 illustrates this.

- Hyperbola \( \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \) or \( \frac{y^2}{a^2} - \frac{x^2}{b^2} = 1 \). The hyperbola never crosses the axis corresponding to the variable having the negative sign. Figures 1.21 and 1.22 illustrate this.

1.6.2 Introduction to Quadric Surfaces

A quadric surface is the graph of a second degree equation in three variables. The general form of such an equation is

\[
Ax^2 + By^2 + Cz^2 + Dxy + Eyz + Gx + Hy + Iz + J = 0
\]

where \(A, B, \ldots, J\) are constants. Quadric surfaces can be classified into five categories: ellipsoids, hyperboloids, cones, paraboloids, and quadric cylinders. In this document, we will study these categories and learn how to identify them. Quadrics surfaces are the 3-D equivalent of conic sections in the plane. You should review conic sections if you are not familiar with them.

For each category, we will learn to find elements which characterize it. These elements include:

1. Standard equation
2. Traces. They are the curves of intersection of the surface with planes parallel to the coordinate planes. Recall the following: (in what follows, \(c\) denotes a constant)
   - A plane parallel to the xy-plane has equation \(z = c\).
Figure 1.21: Hyperbola \( \frac{-x^2}{4} + \frac{y^2}{9} = 1 \)

Figure 1.22: Hyperbola \( \frac{x^2}{4} - \frac{y^2}{9} = 1 \)


A plane parallel to the xz-plane has equation \( y = c \).

A plane parallel to the yz-plane has equation \( x = c \).

3. Intercepts. They are the points of intersection of the surface with the coordinate axes. Recall the following:

- To find the intersection of an object with the x-axis, set \( y = 0 \) and \( z = 0 \) in the equation of the object and solve for \( x \).
- To find the intersection of an object with the y-axis, set \( x = 0 \) and \( z = 0 \) in the equation of the object and solve for \( y \).
- To find the intersection of an object with the z-axis, set \( x = 0 \) and \( y = 0 \) in the equation of the object and solve for \( z \).

1.6.3 Classification of Quadric Surfaces

Quadric surfaces fall into nine categories. To know which category, one has to rewrite it so that it has one of the forms below. Rewriting a quadric surface often involves completing the square. Make sure you remember how to do this.

**Ellipsoid**

It is the 3-D equivalent of an ellipse in 2-D. Its standard equation is of the form:

\[
\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1
\]

For this surface, we will derive in detail the traces and intercepts. For the other surfaces, they will be given and left as an exercise to derive.

- **Traces:** The traces of an ellipsoid are ellipses. This is easy to see. Recall, traces are the curves of intersection of the surface with planes parallel to the coordinate planes. For example, if we try to find the intersection of an ellipsoid with the plane parallel to the xy-plane, that is the plane with equation \( z = k \) for some number \( k \), we obtain:

\[
\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 - \frac{k^2}{c^2}
\]

which is an ellipse as long as \(-c < k < c\). If \( k > c \) or \( k < -c \), the two do not intersect because the equation does not have a solution.

- **Intercepts:** The intercepts of the ellipsoid are \((\pm a, 0, 0), (0, \pm b, 0), (0, 0, \pm c)\). Again, this is easy to see. Recall that intercepts are points of intersection of the surface with the coordinate axes. For example, if we seek the intersection of an ellipsoid with the x-axis, that is when \( y = z = 0 \) then from the equation of an ellipsoid, we obtain \( \frac{x^2}{a^2} = 1 \) that is \( x = \pm a \). Hence the points of intersection are \((\pm a, 0, 0)\). We proceed similarly for the other intercepts.
Example 1.6.1  Consider the ellipsoid \( \frac{x^2}{4} + \frac{y^2}{9} + \frac{z^2}{16} = 1 \).

1. Its graph is shown in Figure 1.23.

2. We illustrate finding its traces by finding the intersection of the ellipsoid with the xy-plane, the plane \( z = 2 \), and the plane \( z = 8 \). Note that the last two planes are parallel to the xy-plane.
   - **Intersection with the xy-plane.** On the xy-plane, \( z = 0 \) hence the equation of the ellipsoid becomes \( \frac{x^2}{4} + \frac{y^2}{9} = 1 \) which is the equation of an ellipse.
   - **Intersection with the plane \( z = 2 \).** When \( z = 2 \), the equation of the ellipsoid becomes
     \[
     \frac{x^2}{4} + \frac{y^2}{9} + \frac{4}{16} = 1 \iff \frac{x^2}{4} + \frac{y^2}{9} = 1 - \frac{1}{4} = \frac{3}{4}
     \]
     This is also the equation of an ellipse. We can put it in standard form by dividing each side by \( \frac{3}{4} \) to obtain
     \[
     \frac{x^2}{3} + \frac{y^2}{27} = 1
     \]
     clearly of the form \( \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \) with \( a = \sqrt{3} \) and \( b = \sqrt{27} \).
   - **Intersection with the plane \( z = 8 \).** When \( z = 8 \), the equation of the ellipsoid becomes
     \[
     \frac{x^2}{4} + \frac{y^2}{9} + \frac{64}{16} = 1 \iff \frac{x^2}{4} + \frac{y^2}{9} + 4 = 1 \iff \frac{x^2}{4} + \frac{y^2}{9} = -3
     \]
     This equation has no solutions since the left side is positive and the right side is strictly negative. This means that the ellipsoid and the plane do not intersect. We could have guessed this without doing any computations. The ellipsoid extends between \(-4\) and \(4\) in the \( z \)-direction. The plane \( z = 8 \) is clearly above the ellipsoid hence never intersects with it.

3. We illustrate finding the intercepts by finding the intersection of the ellipsoid with the x-axis. On the x-axis, both \( y = 0 \) and \( z = 0 \) hence, the equation of the ellipsoid becomes
   \[
   \frac{x^2}{4} = 1 \iff x^2 = 4 \iff x = \pm 2
   \]
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There are two types depending on how many pieces the surface has.

**Hyperboloid of one sheet**

The standard equation is:

\[
\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1
\]

The minus sign could also be in front of the first or second fraction. However, there can only be one minus sign.

In the example shown in Figure 1.24, the minus sign is in front of the \( z \) variable. The axis of the hyperboloid is the \( z \)-axis. The intersection of the hyperboloid with planes perpendicular to the axis of the hyperboloid (the \( xy \)-plane in this case) are ellipses. The intersection with planes parallel to the axis of the hyperboloid are hyperbolas. If, for example we put the minus sign in front of the \( x \) variable, then the axis of the hyperboloid will be the \( x \)-axis. This time, it is the intersection with planes parallel to the \( yz \)-plane which will be ellipses. The graph of such an example is shown in Figure 1.25.

In general, the axis of a hyperboloid of one sheet is the axis corresponding to the variable having the minus sign.

- **Traces:** To see that the intersection of the hyperboloid \( \frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1 \) like the one shown in figure 1.24 with a plane parallel to the \( xy \)-plane is an ellipse, we let \( z = k \) in its equation. We get

\[
\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 + \frac{k^2}{c^2}
\]

which is an ellipse. To see that the intersection of the hyperboloid shown in figure 1.24 with a plane parallel to the \( yz \)-plane is an hyperbola, we let
Figure 1.24: Hyperboloid of one sheet \( \frac{x^2}{4} + \frac{y^2}{9} - \frac{z^2}{16} = 1 \).

\( x = k \) in its equation. We get

\[
\frac{y^2}{b^2} - \frac{z^2}{c^2} = 1 - \frac{k^2}{a^2}
\]

which is a hyperbola.

- **Intercepts:** \( \frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1 \) only has two intercepts: \((\pm a, 0, 0)\), \((0, \pm b, 0)\).

Hyperboloid of two sheets  The standard equation is:

\[
\frac{x^2}{a^2} - \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1
\]

The minus signs could be in front of other fractions also. There has to be **two** minus signs to have a hyperboloid of two sheets. The number of sheets is the same as the number of minus signs. The axis of a hyperboloid of two sheets is the axis corresponding to the variable not having a minus sign.

- **Traces:** The intersection of \( \frac{x^2}{a^2} - \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1 \) with a plane parallel to its axis is a hyperbola. Its intersection with a plane perpendicular to its axis is an ellipse when it exists.

- **Intercepts:** \( \frac{x^2}{a^2} - \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1 \) only has one intercept: \((\pm a, 0, 0)\).

Figure 1.26 shows a hyperboloid of two sheets with axis the x-axis. Figure 1.27 shows a hyperboloid of two sheets with axis the y-axis.
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Figure 1.25: Hyperboloid of one sheet \( -\frac{x^2}{4} + \frac{y^2}{9} + \frac{z^2}{16} = 1 \)

Figure 1.26: Hyperboloid of two sheets \( \frac{x^2}{4} - \frac{y^2}{9} - \frac{z^2}{16} = 1 \)
Elliptic Cones

The standard equation is the same as for a hyperboloid, replacing the 1 on the right side of the equation by a 0. Whether we have one minus sign or two, we get an equation of the form:

\[ \frac{x^2}{a^2} + \frac{y^2}{b^2} = \frac{z^2}{c^2} \]

The axis of the cone corresponds to the variable on the right side of the equation.

Figure 1.29 shows a cone with axis the z-axis. Figure 1.28 shows a cone with axis the y-axis.

Due to the limitations of the graphing software used for this document, it may appear that the graphs shown in Figure 1.29 and Figure 1.28 are made of two pieces. It is not so. If you look at their equations, you will see that they both contain the point \((0, 0, 0)\). Hence, they should be connected.

- **Traces:** The intersection of \( \frac{x^2}{a^2} + \frac{y^2}{b^2} = \frac{z^2}{c^2} \) a plane perpendicular to its axis is an ellipse. The intersection of a cone with a plane parallel to its axis is either a hyperbola if the plane does not contain the origin or a pair of lines if the plane contains the origin.

- **Intercepts:** The origin.

The surface of a cone has the property that if \( P \) is any point on the cone, then the line segment \( OP \) lies entirely on the cone.

Paraboloid

If we start with the equation of a cone, and remove the squares to the expression on the right side of the equal sign, we obtain a paraboloid. There are two types
Figure 1.28: The cone \( \frac{x^2}{4} + \frac{z^2}{9} = \frac{y^2}{16} \)

Figure 1.29: The cone \( \frac{x^2}{4} + \frac{y^2}{9} = \frac{z^2}{16} \)
of paraboloids, depending on whether the expression to the left of the equal sign has a minus sign or not.

**Elliptic paraboloid** The standard equation is
\[
\frac{x^2}{a^2} + \frac{y^2}{b^2} = \frac{z}{c}
\]
Figure 1.30 shows a paraboloid with axis the $z$-axis.

- **Traces:** The intersection \( \frac{x^2}{a^2} + \frac{y^2}{b^2} = \frac{z}{c} \) makes with a plane perpendicular to its axis is an ellipse. The other traces are parabolas.

- **Intercepts:** The origin.

If we change the sign of $c$, the paraboloid is oriented the other way as shown in Figure 1.31. The axis along which the paraboloid extends corresponds to the variable not being squared.

**Hyperbolic paraboloid** The standard equation is:
\[
\frac{x^2}{a^2} - \frac{y^2}{b^2} = \frac{z}{c}
\]
An example is shown in Figure 1.32. This graph is also called a saddle. We will visit it again when we study the problem of finding extrema of functions of several variables.

- **Traces:** The intersection of \( \frac{x^2}{a^2} + \frac{y^2}{b^2} = \frac{z}{c} \) with a plane parallel to the xy-plane is a hyperbola. The intersection with the other coordinate planes is a parabola.
Figure 1.31: Elliptic paraboloid $\frac{x^2}{4} + \frac{y^2}{9} = \frac{-z}{4}$

Figure 1.32: Hyperbolic paraboloid $\frac{x^2}{4} - \frac{y^2}{2} = \frac{z}{4}$
CHAPTER 1. VECTORS AND THE GEOMETRY OF SPACE

Figure 1.33: The elliptic cylinder \( \frac{x^2}{4} + \frac{y^2}{9} = 1 \)

- **Intercepts:** The origin.

Quadric cylinders

There are three types.

**Elliptic cylinder** The standard equation is:

\[
\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1
\]

Figure 1.33 shows such a cylinder. Its axis is the z-axis, the axis corresponding to the variable missing in the equation.

- **Traces:** The intersection of \( \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \) with planes parallel to the xy-plane, that is planes containing the two variables present in the equation, is an ellipse. Its intersection with planes parallel with the other coordinate planes, when they exist, are lines.

- **Intercepts:** The intercepts of \( \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \) are \((\pm a, 0, 0)\) and \((0, \pm b, 0)\)

**Hyperbolic cylinder** The standard equation is:

\[
\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1
\]

Figure 1.34 shows such a cylinder. Its axis is the z-axis, the axis corresponding to the variable missing in the equation.
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Figure 1.34: The hyperbolic cylinder \( \frac{x^2}{4} - \frac{y^2}{9} = 1 \)

- **Traces:** The intersection of \( \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \) with the xy-plane, the plane containing the two variables present in the equation, is a hyperbola. Its intersection with planes parallel with the other coordinate planes, when they exist, are lines.

- **Intercepts:** \( \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \) only has 2 intercepts: \((\pm a, 0, 0)\).

**Parabolic cylinder**  The standard equation is

\[ y = ax^2 \]

Figure 1.35 shows such a cylinder. It appears to be wrapped around the axis corresponding to the missing variable.

- **Traces:** The intersection of \( y = ax^2 \) with the xy-plane, the plane containing the two variables present in the equation, is a parabola. Its intersection with planes parallel with the other coordinate planes, when they exist, are lines.

- **Intercepts:** Every point on the z-axis.

**Conclusion**

We have studied the following forms:
## Classification - Examples

Classifying a quadric surface amounts to transforming its equation until it has one of the forms described above. This often involves "completing the square".

In all the above equations, it is possible to have \((x - h)^2\) instead of \(x^2\), or \((y - k)^2\) instead of \(y^2\) or \((z - l)^2\) instead of \(z^2\). You will recall from pre-calculus that this will simply translate the object, but will not change its shape. For example \(\frac{(x - 1)^2}{2} + \frac{(y + 2)^2}{3} + \frac{z^2}{4} = 1\) has the same shape as \(\frac{x}{2} + \frac{y}{3} + \frac{z^2}{4} = 1\). It is translated from it 1 unit in the positive \(x\) direction and 2 units in the negative \(y\) direction.

### Classifying a Quadric Surface

<table>
<thead>
<tr>
<th>Name</th>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ellipsoid</td>
<td>(\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1)</td>
</tr>
<tr>
<td>Hyperboloid of one sheet</td>
<td>(\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1)</td>
</tr>
<tr>
<td>Hyperboloid of two sheets</td>
<td>(\frac{x^2}{a^2} - \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1)</td>
</tr>
<tr>
<td>Elliptic cone</td>
<td>(\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1)</td>
</tr>
<tr>
<td>Elliptic paraboloid</td>
<td>(\frac{x^2}{a^2} + \frac{y^2}{b^2} = \frac{z^2}{c^2})</td>
</tr>
<tr>
<td>Hyperbolic paraboloid</td>
<td>(\frac{x^2}{a^2} - \frac{y^2}{b^2} = \frac{z^2}{c})</td>
</tr>
<tr>
<td>Elliptic cylinder</td>
<td>(\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1)</td>
</tr>
<tr>
<td>Hyperbolic cylinder</td>
<td>(\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1)</td>
</tr>
<tr>
<td>Parabolic cylinder</td>
<td>(y = ax^2)</td>
</tr>
</tbody>
</table>
Example 1 Classify
\[4x^2 - 9y^2 + z^2 + 36 = 0\]

We only have terms in \(x^2\), \(y^2\), and \(z^2\). We won’t have to complete the square. We simply group these terms together, and put everything else to the right of the equal sign. We obtain:
\[4x^2 - 9y^2 + z^2 = -36\]

Since we want a 1 on the right, we divide by \(-36\), we obtain
\[-\frac{x^2}{9} + \frac{y^2}{4} - \frac{z^2}{36} = 1\]

This is a hyperboloid of two sheets. Its graph is shown in Figure 1.36. Its intersection with a plane parallel to the \(xy\)-plane, that is a plane of coordinates \(z = k\) is
\[-\frac{x^2}{9} + \frac{y^2}{4} = 1 + \frac{k^2}{32}\]

which is a hyperbola. As an exercise, find the other traces.

![Figure 1.36: Graph of \(-\frac{x^2}{9} + \frac{y^2}{4} - \frac{z^2}{36} = 1\)]](image)

Example 2 Classify
\[x^2 + 4y^2 + z^2 - 2x = 0\]
We have a term in \( x^2 \) and \( x \), so we will have to complete the square.

\[
\begin{align*}
x^2 - 2x + 4y^2 + z^2 &= 0 \\
x^2 - 2x + 1 + 4y^2 + z^2 &= 1 \\
(x - 1)^2 + \frac{y^2}{\frac{1}{2}} + z^2 &= 1
\end{align*}
\]

This is an ellipsoid centered at \((1, 0, 0)\). Its intercepts are \((0, 0, 0), (2, 0, 0), \left(0, \pm \frac{1}{2}, 0\right), (0, 0, \pm 1)\).

### 1.6.4 Things to Know

- Know the standard forms of the various quadric surfaces.
- Be able to complete the square to write the equation of a quadric surface in standard form.
- Be able to find the trace (intersection with the coordinate planes) of a quadric surface.

### 1.6.5 Problems

1. Describe and plot \( x^2 + y^2 + 4z^2 = 10 \).
2. Describe and plot \( 9y^2 + z^2 = 16 \).
3. Describe and plot \( x = y^2 - z^2 \).
4. Describe and plot \( x^2 + 2z^2 = 8 \).
5. Describe and plot \( x = z^2 - y^2 \).
6. Describe and plot \( x^2 + 4z^2 = y^2 \).
7. Describe and plot \( x^2 + y^2 = 4 \).
8. Describe and plot \( x^2 + 4z^2 = 16 \).
9. Describe and plot \( 9x^2 + y^2 + z^2 = 9 \).
10. Describe and plot \( 4x^2 + 9y^2 + 4z^2 = 36 \).
11. Describe and plot \( z = x^2 + 4y^2 \).
12. Describe and plot \( x = 4 - 4y^2 - z^2 \).
13. Describe and plot \( x^2 + y^2 = z^2 \).
14. Describe and plot \( x^2 + y^2 - z^2 = 1 \).
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15. Describe and plot \( z^2 - x^2 - y^2 = 1 \).
16. Describe and plot \( y^2 - x^2 = z \).
17. Describe and plot \( z = 1 + y^2 - x^2 \).
18. Describe and plot \( y = - (x^2 + z^2) \).
19. Describe and plot \( x^2 + z^2 = 1 \).
20. Describe and plot \( z = - (x^2 + y^2) \).
21. Describe and plot \( 4y^2 + z^2 - 4x^2 = 4 \).
22. Describe and plot \( x^2 + y^2 = 1 - \frac{z^2}{25} \).
23. Describe and plot \( 5x^2 = z^2 - 3y^2 \).
24. Describe and plot \( x^2 - 1 = \frac{y^2}{16} + \frac{z^2}{2} \).
25. Write \( 4x^2 - 8x - y^2 - 4y - 4z^2 + 24z - 52 = 0 \) in standard form, then describe it and plot it.

26. Derive the traces and intercepts for each of the quadric surfaces.

(a) Ellipsoid: \( \frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1 \)
(b) Hyperboloid of one sheet: \( \frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1 \)
(c) Hyperboloid of two sheets: \( \frac{x^2}{a^2} - \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1 \)
(d) Elliptic cone: \( \frac{x^2}{a^2} + \frac{y^2}{b^2} = \frac{z^2}{c^2} \)
(e) Elliptic paraboloid: \( \frac{x^2}{a^2} + \frac{y^2}{b^2} = \frac{z}{c} \)
(f) Hyperbolic paraboloid (saddle): \( \frac{x^2}{a^2} - \frac{y^2}{b^2} = \frac{z}{c} \)
(g) Elliptic cylinder: \( \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \)
(h) Hyperbolic cylinder: \( \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \)
(i) Parabolic cylinder: \( y = ax^2 \)
1.6.6 Answers

1. Describe and plot $x^2 + y^2 + 4z^2 = 10$.

This can be written as $\frac{x^2}{10} + \frac{y^2}{10} + \frac{z^2}{2} = 1$ which is an ellipsoid whose cross section in the $xy$-plane is a circle.

2. Describe and plot $9y^2 + z^2 = 16$.

This can be written as $\frac{y^2}{\frac{16}{9}} + \frac{z^2}{16} = 1$ which is an elliptic cylinder along the $x$-axis.
3. Describe and plot \( x = y^2 - z^2 \)
This is a hyperbolic paraboloid (saddle) along the \( x \)-axis.

4. Describe and plot \( x^2 + 2z^2 = 8 \).
This can be written as \( \frac{x^2}{8} + \frac{z^2}{4} = 1 \) which is an elliptic cylinder along the \( y \)-axis.
5. Describe and plot \( x = z^2 - y^2 \).
This is a hyperbolic paraboloid or a saddle, we would sit along the \( z \)-axis, legs along \( y \)-axis.

6. Describe and plot \( x^2 + 4z^2 = y^2 \).
This is an elliptic cone along the \( y \)-axis:
7. Describe and plot $x^2 + y^2 = 4$.
   This is a cylinder along the z-axis.

8. Describe and plot $x^2 + 4z^2 = 16$.
   This can be written as $\frac{x^2}{4^2} + \frac{z^2}{2^2} = 1$, hence it is an elliptic cylinder along
the y-axis.

   This can be written as $x^2 + \frac{y^2}{3^2} + \frac{z^2}{3^2} = 1$, hence it is an ellipsoid.
10. Describe and plot \(4x^2 + 9y^2 + 4z^2 = 36\).
   This can be written as \(\frac{x^2}{3^2} + \frac{y^2}{2^2} + \frac{z^2}{3^2} = 1\), hence it is an ellipsoid.

11. Describe and plot \(z = x^2 + 4y^2\).
    This is an elliptic paraboloid along the z-axis.
12. Describe and plot $x = 4 - 4y^2 - z^2$.

This can be written as $-(x - 4) = 4y^2 + z^2$, hence it is an elliptic paraboloid along the x-axis, with vertex at $(4, 0, 0)$.

13. Describe and plot $x^2 + y^2 = z^2$.

This is an elliptic cone along the z-axis.
14. Describe and plot $x^2 + y^2 - z^2 = 1$. This is a hyperboloid of one sheet, along the $z$-axis.

15. Describe and plot $z^2 - x^2 - y^2 = 1$. This is a hyperboloid of two sheets.
16. Describe and plot \( y^2 - x^2 = z \).
   This is a hyperbolic paraboloid (saddle), centered at the origin.

17. Describe and plot \( z = 1 + y^2 - x^2 \).
   This can be written as \( z - 1 = y^2 - x^2 \), hence it is a hyperbolic paraboloid (saddle), centered at \((0, 0, 1)\).
18. Describe and plot \( y = - (x^2 + z^2) \).
   This is an elliptic paraboloid along the y-axis.

![](image1.png)

19. Describe and plot \( x^2 + z^2 = 1 \).
   This is a cylinder along the y-axis.

![](image2.png)
20. Describe and plot \( z = - (x^2 + y^2) \).
This is an elliptic paraboloid along the z-axis.

21. Describe and plot \( 4y^2 + z^2 - 4x^2 = 4 \).
This can be written as \( y^2 + \frac{z^2}{4} - x^2 = 1 \), hence it is a hyperboloid of one sheet.
22. Describe and plot \( \frac{x^2}{9} + \frac{y^2}{36} = 1 - \frac{z^2}{25} \).
   Ellipsoid

23. Describe and plot \( 5x^2 = z^2 - 3y^2 \).
   Elliptic cone along \( z \)-axis.
24. Describe and plot \( \frac{x^2}{9} - 1 = \frac{y^2}{16} + \frac{z^2}{2} \).
Hyperboloid of 2 sheets with \( x \)-axis going through.

25. Write \( 4x^2 - 8x - y^2 - 4y - 4z^2 + 24z - 52 = 0 \) in standard form, then describe it and plot it.
The standard form is: \[
\frac{(x - 1)^2}{4} - \frac{(y + 2)^2}{16} - \frac{(z - 3)^2}{4} = 1
\]
hence this is a hyperboloid of 2 sheets with \(x\)-axis going through, centered at \((1, -2, 3)\).

26. Derive the traces and intercepts for each of the quadric surfaces.
The answers are in the notes.
Bibliography

