Optimization Problems

1. A farmer with 2400 ft of fencing wants to construct a rectangular pen that will border a straight river. He needs no fence along the river. What are the dimensions of the pen of largest of area that the farmer can build?

2. A farmer with 750 ft of fence wants to enclose a rectangular area and then divide it into four pens with fencing parallel to one side of the rectangle. What is the largest possible total area of the four pens? Also, what should be the dimensions of the pen in order that this maximum be achieved?

3. A box with an open top is to be constructed from a square piece of cardboard 3 ft wide by cutting out a square from each of the four corners and bending up the sides. Find the largest volume that such a box can have and determine what size squares should be cut from the corners in order that this maximum volume can be achieved.

4. A box with an open top is to be constructed from a rectangular piece of cardboard with dimensions 4 ft × 8 ft by cutting out a square from each of the four corners and bending up the sides. Find the largest volume that such a box can have and determine what size squares should be cut from the corners in order that this maximum volume can be achieved.

5. Determine the production level that will maximize the profit for a company with cost and demand functions

\[ C(x) = 0.00007x^3 - 0.01x^2 + 1.26x + 84 \]

and

\[ p(x) = -0.01x + 3.5. \]

6. A store has been selling 200 compact disc players a week at $350 each. A market survey indicates that for each $10 rebate offered to the buyers, the number of cd players sold will increase by 20 per week. Find the demand function and the revenue function. How large a rebate should the store offer in order to maximize its revenue?
7. The cost, in dollars, of producing \( x \) yards of a certain fabric is

\[
C(x) = 0.0005x^3 - 0.1x^2 + 12x + 1200
\]

and the company finds that if it sells \( x \) yards, it can charge

\[
p(x) = -0.00021x + 29
\]

dollars per yard for the fabric.

(a) Find the profit function.
(b) Use calculus to find the production level for maximum profit.

8. An aircraft manufacturer wants to determine the best selling price for a new airplane. The company estimates that the initial cost of designing the airplane and setting up factories in which to build it will be 500 million dollars, and that the additional cost of manufacturing each plane can be modelled by the function

\[
m(x) = 0.01x^2 + 15x
\]

where \( x \) is the number of aircraft produced and \( m(x) \) is the manufacturing cost in millions of dollars. The company estimates that if it charges a price \( p \) (in millions of dollars) for each plane, it will be able to sell

\[
x = -7.7p + 320
\]

planes.

(a) Find the cost, price, revenue, and profit functions for this aircraft.
(b) Find the production level and the associated selling price of the aircraft that maximizes profit.

9. A baseball team plays in a stadium that holds 55,000 spectators. With ticket prices set at $10, the average attendance had been 27,000. When ticket prices were lowered to $8, the average attendance rose to 33,000.

(a) Find the demand function (assuming that it is linear).
(b) At what price should tickets be sold in order to maximize revenue?
10. During the summer months, Terry makes and sells necklaces on the beach. Last summer, he sold the necklaces for $10 each and his sales averaged 20 per day. When he increased the price by $1, he found that he lost two sales per day.

(a) Find the demand function, assuming that it is linear.

(b) If the material for each necklace costs Terry $6, how should he set the selling price in order to maximize profit?

11. A manufacturer has been selling 1000 television sets a week at $450 each. A market survey indicates that for each $10 rebate offered to the buyer, the number of sets sold will increase by 100 per week.

(a) Find the demand function.

(b) How large a rebate should the company offer the buyer in order to maximize its revenue?

(c) If its weekly cost function is $C(x) = 150x + 68,000$, how should it set the size of the rebate in order to maximize its profit?

12. The manager of a 100 unit apartment complex knows from experience that all units will be occupied if the rent is $400 per month. A market survey suggests that, on average, one additional unit will remain vacant for each $5 increase in rent. What rent should the manager charge in order to maximize revenue?

13. A hockey team plays in an arena with a seating capacity of 15,000 spectators. With the ticket price set at $12, average attendance at games has been 11,000. A market survey indicates that for each dollar that the price is lowered, average attendance will increase by 1000. How should the owners of the team set the ticket price in order to maximize revenue from ticket sales?

14. A manufacturer determines that the cost of making $x$ units of a commodity is

$$C(x) = 0.001x^3 - 0.2x^2 + 25x + 1800$$

and the demand function is

$$p(x) = -0.03x + 48.2.$$
(a) Find the profit function for the commodity.
(b) Use calculus to find the production level for maximum profit.