Sets, Relations, and Functions

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Abstract

We give definitions of the concepts of Set, Relation, and Function, and look at some examples.

1 Sets

A set is a well–defined collection of objects.

An example of a set is the set, $A$, defined by

$$A = \{1, 2, 5, 10\}.$$

The set $A$ has four members (also called elements). The members of $A$ are the numbers 1, 2, 5, and 10.

Another example of a set is the set, $B$, defined by

$$B = \{\text{Arkansas, Hawaii, Michigan}\}.$$

The set $B$ has three members – the states Arkansas, Hawaii, and Michigan. In this course, we will restrict our attention to sets whose members are real numbers or ordered pairs of real numbers. An example of a set whose members are ordered pairs of real numbers is the set, $C$, defined by

$$C = \{(6, 8), (-4, 7), (5, -1), (10, 10)\}.$$

Note that the set $C$ has four members.

When describing a set, we never list any of its members more than once. Thus, the set $\{1, 2, 5, 5, 10\}$ is the same as the set $\{1, 2, 5, 10\}$. Actually, it is
not even correct to write this set as \{1, 2, 5, 5, 10\} because, in doing so, we are listing one of the members more than once.

If an object, \(x\), is a member of a set, \(A\), then we write
\[x \in A.\]
This notation is read as “\(x\) is a member of \(A\)”, or as “\(x\) is an element of \(A\)” or as “\(x\) belongs to \(A\)”.

If the object, \(x\), is not a member of the set \(A\), then we write
\[x \notin A.\]

When we say that a set is a “well-defined” collection of objects, we mean that given any object, \(x\), we are able to determine whether \(x \in A\) or \(x \notin A\).

Here are some examples that refer to the sets \(A\), \(B\), and \(C\) defined above:

- \(2 \in A\)
- \(4 \notin A\)
- \(\text{Georgia} \notin B\)
- \((5, -1) \in C\)
- \(5 \notin C\).

1.1 Finite Versus Infinite Sets

A finite set is a set that has only finitely many members. For example, the sets \(A\), \(B\), and \(C\) given above are all finite sets. (\(A\) and \(C\) each have four members and \(B\) has three members.)

An infinite set is a set that has infinitely many members. For example, the set
\[N = \{1, 2, 3, \ldots\}\]
is an infinite set, because every positive integer is a member of \(N\). We cannot possibly list every member of \(N\). When we write \(1, 2, 3, \ldots\), this means that \(N\) contains the numbers \(1, 2, 3\), and all numbers continuing in this pattern without stopping (which would mean all positive integers).

The empty set, denoted by \(\emptyset\), is the set with no members.

1.2 Subsets, Unions, and Intersections of Sets

If \(A\) and \(B\) are sets and if every member of \(A\) is also a member of \(B\), then we say that \(A\) is a subset of \(B\) and write \(A \subseteq B\).
If \( A \subseteq B \) and \( B \subseteq A \), then we say that \( A \) equals \( B \) and we write \( A = B \). If \( A \subseteq B \) but \( B \nsubseteq A \), then we say that \( A \) is a proper subset of \( B \) and we write \( A \subset B \).

To see some examples of these ideas, consider the sets

\[
A = \{1, 2, 5, 10\} \\
B = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11\} \\
C = \{1, 2, 5, 10\}.
\]

For these sets, all of the following statements are true:

\[
A \subseteq B \\
A \subset B \\
B \nsubseteq A \\
A \neq B \\
A \subseteq C \\
C \subseteq A \\
A = C.
\]

If \( A \) and \( B \) are two sets, then the union of \( A \) and \( B \), denoted by \( A \cup B \), is defined to be the set whose members are all of the members of \( A \) together with all of the members of \( B \). In other words,

\[
x \in A \cup B \text{ if and only if } x \in A \text{ or } x \in B.
\]

The intersection of the sets \( A \) and \( B \), denoted by \( A \cap B \), is defined to be the set whose members are the common members of \( A \) and \( B \). In other words,

\[
x \in A \cap B \text{ if and only if } x \in A \text{ and } x \in B.
\]

In order to illustrate these ideas, let us consider the sets

\[
A = \{1, 2, 5, 10\} \\
B = \{1, 2, 4, 11\}.
\]

For these sets, we have

\[
A \cup B = \{1, 2, 4, 5, 10, 11\} \\
A \cap B = \{1, 2\}.
\]
If two sets, $A$ and $B$, have no members in common, then $A \cap B = \emptyset$ and we say that the sets $A$ and $B$ are **disjoint**. An example of two disjoint sets are

$$A = \{1, 2, 5, 10\}$$

$$B = \{0, 3, 4, 11\}.$$ 

### 1.3 Interval Notation

A **finite interval**, $I$, is a set consisting of all real numbers that lie between two given real numbers, $a$ and $b$. The interval, $I$, might or might not have the numbers $a$ and $b$ as members, but it definitely contains all of the numbers that lie between $a$ and $b$. An example of a finite interval is

$$I = [5, 11).$$

This set consists of all real numbers that lie between the numbers 5 and 11. The square bracket on the left means that the number 5 is a member of $I$. The rounded bracket on the right means that the number 11 is **not** a member of $I$. Thus, some true statements about $I$ are

$$5 \in I$$

$$11 \notin I$$

$$8.69 \in I$$

$$10.9999994 \in I$$

$$4.98 \notin I.$$ 

We can draw a picture of the interval $I = [5, 11)$ as follows:
Note that $I = [5, 11)$ is an infinite set. Thus, when we say that $I$ is a finite interval, the word “finite” should not be mistaken to mean that $I$ has only a finite number of members. (It does not!) We call $I$ a finite interval because it “ends” on both the left and the right. This is in contrast to an infinite interval, which we are about to define. Before defining what an infinite interval is, let us list the different possible types of finite intervals. For each of these, $a$ and $b$ are assumed to be two given real numbers with $a < b$:

<table>
<thead>
<tr>
<th>interval</th>
<th>description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$[a, b]$</td>
<td>contains $a$ and $b$ and all real numbers between $a$ and $b$</td>
</tr>
<tr>
<td>$(a, b)$</td>
<td>contains all real numbers between $a$ and $b$, but contains neither $a$ nor $b$</td>
</tr>
<tr>
<td>$[a, b)$</td>
<td>contains $a$ and all real numbers between $a$ and $b$, but does not contain $b$</td>
</tr>
<tr>
<td>$(a, b]$</td>
<td>contains $b$ and all real numbers between $a$ and $b$, but does not contain $a$</td>
</tr>
</tbody>
</table>

An infinite interval is a set that contains all real numbers that are greater than some given real number $a$, or a set that contains all real numbers that are less than some given real number $a$, or a set that contains all real numbers. In the first two cases mentioned, the interval may or may not contain the number $a$ itself. In the last case mentioned, the interval is simply the entire set of real numbers.

An example of an infinite interval is

$I = [5, \infty)$. 

The symbol “$\infty$” is called “infinity”. It is not a real number, but is used here as a notational convenience, indicating that the interval $I$ “starts” at 5 and goes on forever to the right on the number line without stopping. A picture of the interval $I = [5, \infty)$ is shown below:
Some true statements about the set $I = [5, \infty)$ are

\begin{itemize}
  \item $5 \in I$
  \item $5.000001 \in I$
  \item $148.6578 \in I$
  \item $4.98 \notin I$
\end{itemize}

The five possible types of infinite intervals are listed below. In these listings, $a$ is assumed to be some given real number.

<table>
<thead>
<tr>
<th>interval</th>
<th>description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$[a, \infty)$</td>
<td>contains $a$ and all real numbers greater than $a$</td>
</tr>
<tr>
<td>$(a, \infty)$</td>
<td>contains all real numbers greater than $a$, but does not contain $a$</td>
</tr>
<tr>
<td>$(-\infty, a]$</td>
<td>contains $a$ and all real numbers less than $a$</td>
</tr>
<tr>
<td>$(-\infty, a)$</td>
<td>contains all real numbers less than $a$, but does not contain $a$</td>
</tr>
<tr>
<td>$(-\infty, \infty)$</td>
<td>contains all real numbers</td>
</tr>
</tbody>
</table>

Let us once again stress the two different meaning of the words “finite” and the two different meanings of the word “infinite” that we use when referring to intervals: A finite interval is an interval that “stops” both on the left and on the right. An infinite interval is an interval that either does not stop on the left or does not stop on the right (or both). All intervals, whether they be finite or infinite intervals, are infinite sets (because any interval has infinitely many members).

### 1.4 Exercises

1. Let $A$, $B$, $C$, and $D$ be the following sets:

   $A = \{-4, -2, 0, 1.6, \sqrt{3}\}$
   $B = \{0, 1, 2, 3, \ldots\}$
   $C = [-2, 2]$
   $D = [1, \infty)$

   (a) Which of the above sets have the number $-2$ as a member?
   (b) Which of the above sets have the number $1$ as a member?
   (c) Which of the above sets have the number $1.73$ as a member?
(d) Which of the above sets have the number 468 as a member?
(e) Which of the above sets have the number 468.5 as a member?
(f) Which of the above sets are finite sets?
(g) Which of the above sets are infinite sets?
(h) Which of the above sets are intervals?
(i) Which of the above sets are finite intervals?
(j) Which of the above sets are infinite intervals?

2. Draw pictures of the sets $A$, $B$, $C$, and $D$ on the number line.

3. Determine (meaning write descriptions of) the sets $A \cup B$, $A \cup C$, $A \cup D$, $B \cup C$, $B \cup D$, and $C \cup D$. Also, draw pictures of each of these sets on the number line and state whether each set is a finite set or an infinite set. The first one is done as an example:

$$A \cup B = \{-4, -2, 1.6, \sqrt{3}\} \cup \{0, 1, 2, 3, \ldots\}.$$ 

There is really no “nicer” way to write a description of $A \cup B$. (By “nicer”, we mean a description that does not use the union symbol.) A picture of $A \cup B$ on the number line is shown below.

![A Picture of $A \cup B$](image)

$A \cup B$ is an infinite set.

4. Determine (meaning write descriptions of) the sets $A \cap B$, $A \cap C$, $A \cap D$, $B \cap C$, $B \cap D$, and $C \cap D$. Also, draw pictures of each of these sets on the number line. The first one is done as an example:

$$A \cap B = \{0\}$$
A ∩ B is a finite set. (It has only one member.)

5. Suppose that $A$ is any set. Is the statement $A \subseteq A$ true? Is the statement $A \subset A$ true? Explain your answers.

6. Suppose that $A$ and $B$ are sets and that $A \subseteq B$. What is $A \cup B$ in this case? What is $A \cap B$?

7. Consider the following infinite number of sets (each of which is a finite interval):

$$A_1 = [0, 1]$$
$$A_2 = \left[0, \frac{1}{2}\right]$$
$$A_3 = \left[0, \frac{1}{3}\right]$$

: 

Find $A_1 \cap A_2$, $A_1 \cap A_2 \cap A_3$, and $A_1 \cap A_2 \cap A_3 \cap A_4$. What can you say about the infinite intersection $A_1 \cap A_2 \cap A_3 \cap A_4 \cap \cdots$?

8. Repeat exercise 7, except take the sets in question to be

$$A_1 = (0, 1]$$
$$A_2 = \left(0, \frac{1}{2}\right]$$
$$A_3 = \left(0, \frac{1}{3}\right]$$

:
2 Relations

A relation is a set whose members are ordered pairs of real numbers. An example of a relation is

\[ f = \{(1, 2), (2, -1), (-1, 6), (1, 4)\}. \]

We can get a visual picture of a relation by graphing the points that belong to the relation on an \(x, y\) grid. Here is a picture (also called the graph) of the relation \(f\) defined above.

If \(f\) is a relation, then the domain of \(f\), denoted by \(\text{domain}(f)\), is defined to be the set of all first components of \(f\), and the range of \(f\), denoted by \(\text{range}(f)\), is defined to be the set of all second components of \(f\). Thus, for example, the relation

\[ f = \{(1, 2), (2, -1), (-1, 6), (1, 4)\} \]

has domain

\[ \text{domain}(f) = \{-1, 1, 2\} \]

and range

\[ \text{range}(f) = \{-1, 2, 4, 6\}. \]
2.1 Relations Defined Via Equations

If we say that a relation, \( f \), is defined by an equation, such as

\[ x^2 + y^2 = 1, \]

then we mean that the relation consists of all ordered pairs of real numbers, \((x, y)\), that satisfy the equation. Thus, for example, the relation defined by the above equation has the ordered pair \((1, 0)\) as a member because

\[ 1^2 + 0^2 = 1. \]

This relation also has the ordered pair \((-\frac{1}{2}, \frac{\sqrt{3}}{2})\) as a member because

\[ \left(-\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2 = 1. \]

However, the ordered pair \((\frac{1}{2}, \frac{1}{2})\) is not a member of this relation because

\[ \left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2 \neq 1. \]

The graph of the relation, \( f \), defined by the equation \( x^2 + y^2 = 1 \) is shown below. This graph is the unit circle (the circle with radius 1 centered at the point \((0, 0)\) in the \(x, y\) plane).
The relation
\[ f = \{(x, y) \mid x^2 + y^2 = 1\} \]
has domain
\[ \text{domain} (f) = [-1, 1] \]
and range
\[ \text{range} (f) = [-1, 1]. \]

Sometimes, in defining a relation via an equation, it is desirable to restrict the domain and/or the range. Here is an example: Let \( g \) be the relation defined by the equation
\[ x^2 + y^2 = 1 \]
with the restriction that
\[ y \geq 0. \]

A succinct way to write this is
\[ g = \{(x, y) \mid x^2 + y^2 = 1 \text{ and } y \geq 0\}. \]

Since the relation \( g \) has the restriction \( y \geq 0 \), we see that \( g \) contains no ordered pair whose second component is a negative number. For example, the ordered pair \((0, -1)\) is not a member of \( g \) even though \((0)^2 + (-1)^2 = 1\), because it is not true that \(-1 \geq 0\). The graph of the relation \( g \) is shown below. This graph is the upper half of the unit circle.

![Graph of the relation g](image)

The domain of \( g \) is
\[ \text{domain} (g) = [-1, 1] \]
and the range of \( g \) is
\[ \text{range} (g) = [0, 1]. \]
2.2 More Examples of Relations

Consider the relation

\[ f = \{ (x, y) \mid y = x^2 \} . \]

To find some specific pairs in this relation, we can pick any number we like for \( x \) and then compute the corresponding value of \( y \), since the relation is defined by the equation \( y = x^2 \). Some pairs in the relation \( f \) are \((0, 0), (1, 1), (2, 4), (3, 9), (2.5, 6.25), (−1, 1), (−2, 4), (−\sqrt{7}, 7)\), etc. A graph of the relation \( f \) is shown below. The graph is a parabola with vertex at the point \((0, 0)\).

![Graph of f](image)

Note that the domain of \( f \) is

\[ \text{domain}(f) = (-\infty, \infty) \]

because all real numbers appear as first components of members of \( f \). However, the range of \( f \) is

\[ \text{range}(f) = [0, \infty) \]

because only nonnegative numbers appear as second components of members of \( f \).

As another example, consider the relations \( f \) and \( g \) defined by

\[ f = \{ (x, y) \mid y = \sqrt{x} \} \]

and

\[ g = \{ (x, y) \mid y^2 = x \} \]

The domain of \( f \) is

\[ \text{domain}(f) = [0, \infty) \]
because if \( x \) is a negative number, then \( \sqrt{x} \) is not a real number. Thus, if \( x \) is a negative number, then there is no real number, \( y \), such that \( y = \sqrt{x} \).

Some pairs of \( f \) are \((0, 0), (1, 1), (4, 2), (9, 3), (7, \sqrt{7})\), etc. The graph of \( f \) is shown below.

![Graph of \( f \)](image)

\[
f = \{(x, y) \mid y = \sqrt{x}\}
\]

Observe that the range of \( f \) is

\[
\text{range} \ (f) = [0, \infty).
\]

Let us now compare \( f \) to the relation

\[
g = \{(x, y) \mid y^2 = x\}.
\]

If a pair of real numbers, \((x, y)\), satisfies the equation \( y = \sqrt{x} \), then this pair also satisfies the equation \( y^2 = x \). However, not every pair, \((x, y)\), that satisfies \( y^2 = x \) also satisfies \( y = \sqrt{x} \) (since it could be the case that \( y = -\sqrt{x} \)). This means that every member of \( f \) is also a member of \( g \), but not every member of \( g \) is a member of \( f \). For example, the pair \((4, 2)\) is a member of both relations \( f \) and \( g \). However, the pair \((4, -2)\) is a member of \( g \) but not a member of \( f \). The graph of \( g \) is shown below.
The domain of $g$ is $\text{domain}(g) = [0, \infty)$ and the range of $g$ is $\text{range}(g) = (-\infty, \infty)$.

2.3 Exercises

1. Graph the following relations and state the domain and range of each.

(a) $f = \{(−3, 0), (−2, −2), (−1, 5), (−1, 0), (0, 4), (2, 4), (2, 2)\}$

(b) $f = \{(−3, 0), (−2, −2), (−1, 0), (0, 4), (2, 4)\}$

(c) $f = \{(−3, 4), (−2, 4), (−1, 4), (0, 4), (2, 4)\}$

(d) $f = \{(x, y) \mid y = 4\}$

(e) $f = \{(x, y) \mid x = 3\}$

(f) $f = \{(x, y) \mid y = x\}$
(g) \[ f = \{(x, y) \mid y = x^3\} \]

(h) \[ f = \{(x, y) \mid y = |x|\} \]

(i) \[ f = \{(x, y) \mid x^2 + y^2 = 1 \text{ and } x \geq 0\} \]

(j) \[ f = \{(x, y) \mid y = x^2 \text{ and } x \geq 0\} \]

(k) \[ f = \{(x, y) \mid y = x^2 \text{ and } x \leq 0\} \]

(l) \[ f = \{(x, y) \mid y = x^2 \text{ and } x \in [-1, 2)\} \]

(m) \[ f = \{(x, y) \mid |x| + |y| = 1\} \]

**Hint:** First find all members, \((x, y)\), of \(f\) for which \(x \geq 0\) and \(y \geq 0\). Then find all members for which \(x \leq 0\) and \(y \geq 0\), etc.

(n) \[ f = \{(x, y) \mid |x| + y = 1\} \]
3 Functions

A function, \( f \), is a relation in which no two pairs have the same first component. For example,

\[
 f = \{(1, 1), (-3, 1), (2, 4), (3, -2)\}
\]

is a function because no two of its pairs have the same first component, but

\[
 g = \{(1, 1), (-3, 1), (2, 4), (2, -2)\}
\]

is not a function, because the pairs \((2, 4)\) and \((2, -2)\) have the same first component.

The graphs of the relations \( f \) and \( g \) are shown below.

\[
 f = \{(1, 1), (-3, 1), (2, 4), (3, -2)\}
\]

\[
 g = \{(1, 1), (-3, 1), (2, 4), (2, -2)\}
\]
3.1 The “Vertical Line” Test

If we look at the graph of a function, we notice that any vertical line that we draw can intersect the graph of the function at most one time. This is because no two points on the graph of the function have the same first component. On the other hand, if we look at the graph of a relation that is not a function, then we can draw at least one vertical line that will intersect the graph more than once. This is because the graph of such a relation contains at least two points that have the same first component. We thus have what is usually called the “Vertical Line Test” for determining whether or not a given relation is a function:

**Vertical Line Test:** If $f$ is a relation and no vertical line intersects the graph of $f$ more than once, then $f$ is a function. If, however, there is at least one vertical line that intersects the graph of $f$ more than once, then $f$ is not a function.

We now give some examples of relations that are functions and some relations that are not functions. These conclusions can be reached by studying the graphs of each of these relations, all of which appear earlier in this document.

<table>
<thead>
<tr>
<th>Relation</th>
<th>Is or Is not a Function</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f = {(1, 2), (2, -1), (-1, 6), (1, 4)}$</td>
<td>is not a function</td>
</tr>
<tr>
<td>$f = {(x, y) \mid x^2 + y^2 = 1}$</td>
<td>is not a function</td>
</tr>
<tr>
<td>$g = {(x, y) \mid x^2 + y^2 = 1 \text{ and } y \geq 0}$</td>
<td>is a function</td>
</tr>
<tr>
<td>$f = {(x, y) \mid y = x^2}$</td>
<td>is a function</td>
</tr>
<tr>
<td>$f = {(x, y) \mid y = \sqrt{x}}$</td>
<td>is a function</td>
</tr>
<tr>
<td>$g = {(x, y) \mid y^2 = x}$</td>
<td>is not a function</td>
</tr>
<tr>
<td>$f = {(1, 1), (-3, 1), (2, 4), (3, -2)}$</td>
<td>is a function</td>
</tr>
<tr>
<td>$g = {(1, 1), (-3, 1), (2, 4), (2, -2)}$</td>
<td>is not a function</td>
</tr>
</tbody>
</table>

3.2 Some Notation Pertaining to Functions

If $f$ is a function, then every number, $x$, in the domain of $f$ corresponds to exactly one number, $y$, in the range of $f$. If $y$ is the number in the range of $f$ that corresponds to the number, $x$, in the domain of $f$, then we write $f(x) = y$. This is standard notation that appears in almost all mathematical
writing. In fact, for functions that are defined via formulas, we often just write

\[ y = f(x) \]

instead of writing something like

\[ f = \{(x, y) \mid \text{equation}\} \]

Let us look at three examples that illustrate the above notation:

First, suppose that \( f \) is the function

\[ f = \{(1, 1), (-3, 1), (2, 4), (3, -2)\} . \]

Then we can write

\[
\begin{align*}
  f(1) &= 1 \\
  f(-3) &= 1 \\
  f(2) &= 4 \\
  f(3) &= -2.
\end{align*}
\]

As a second example, suppose that \( f \) is the function

\[ f = \{(x, y) \mid y = x^2\} . \]

An alternative way to describe this function is by simply writing

\[ f(x) = x^2 \]

If we wish to find, for example, the \( y \) value that corresponds to \( x = -3.4 \), we compute

\[ f(-3.4) = (-3.4)^2 = 11.56. \]

As our third example, suppose that \( g \) is the function

\[ g = \{(x, y) \mid x^2 + y^2 = 1 \text{ and } y \geq 0\} \]

If we solve the equation \( x^2 + y^2 = 1 \) for \( y \), then we obtain

\[ y = \pm\sqrt{1 - x^2} \]

and the restriction \( y \geq 0 \) implies that

\[ y = \sqrt{1 - x^2} . \]
Thus, we can write

\[ g(x) = \sqrt{1 - x^2} \]

If we wish to find, for example, the \( y \) value that corresponds to \( x = -\frac{1}{2} \), we compute

\[ g \left( -\frac{1}{2} \right) = \sqrt{1 - \left( -\frac{1}{2} \right)^2} = \sqrt{1 - \frac{1}{4}} = \sqrt{\frac{3}{4}} = \frac{\sqrt{3}}{2}. \]

The figure below shows the point \( \left( -\frac{1}{2}, \frac{\sqrt{3}}{2} \right) \) on the graph of \( g \).

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3.3 A Word of Caution

In using the shorter “\( f(x) \)” notation to describe a function, we need to avoid confusion that might arise if a restricted domain is intended. For example, if we wish to study the function

\[ f = \{(x, y) \mid y = x^2\} \]

(with no restriction on the domain of \( f \)), then it is okay to describe this function as

\[ f(x) = x^2 \]

and no confusion should arise. However, suppose that we wish to study the function

\[ f = \{(x, y) \mid y = x^2 \text{ and } x \geq 0\} \]

19
In this case, if we wish to use the shorter “$f(x)$” notation, we should be more specific by writing

$$f(x) = x^2, \quad x \geq 0$$
or

$$f(x) = x^2, \quad x \in [0, \infty).$$

We will make the convention that if we describe a function by writing

$$f(x) = \text{formula},$$

then the domain of $f$ will be assumed to be the largest possible set of real numbers, $x$, for which the formula defining $f$ yields a real number. If we wish to use some more restricted domain, $D$, then we will write

$$f(x) = \text{formula}, \quad x \in D.$$  

For example, if we write

$$f(x) = \frac{1}{\sqrt{x} - 3},$$

then it will be understood that $f$ is the function defined by the equation

$$y = \frac{1}{\sqrt{x} - 3}$$

with domain

$$\text{domain}(f) = [0, 9) \cup (9, \infty),$$
because $[0, 9) \cup (9, \infty)$ is the largest possible set of real numbers for which the formula is defined\(^1\). If, however, we wish to study the function defined by the same formula with a restricted domain, say $D = (9, \infty)$, then we would write

$$f(x) = \frac{1}{\sqrt{x} - 3}, \quad x \in (9, \infty)$$

or

$$f(x) = \frac{1}{\sqrt{x} - 3}, \quad 9 < x < \infty.$$  

\(^1\)Note that the formula $y = 1/(\sqrt{x} - 3)$ is not defined for any negative number, $x$, because there is a $\sqrt{x}$ in the formula. The formula is also not defined for $x = 9$, because $x = 9$ makes the denominator of the formula be 0.
3.4 Exercises

1. Decide which of the relations given in Exercise 2.3 are functions and which are not functions.

2. For those relations in Exercise 2.3 that are functions, describe these functions in the form

   \[ f(x) = \text{formula}, \quad x \in D \]

   (where the “\(x \in D\)” part might or might not be necessary).

3. What is the (implied) domain of the function

   \[ f(x) = \frac{1}{x} \]

   Compute the values of \(f(1), f(2), f(4), f(10), f(100), f(1000), f(1/2), f(1/4), f(1/10), f(1/100), f(1/1000), f(-1), f(-2), f(-4), f(-10), f(-100), f(-1000), f(-1/2), f(-1/4), f(-1/10), f(-1/100), f(-1/1000)\) and then plot the corresponding points on the graph of \(f\). Use these points as guidance in constructing a complete graph of \(f\).