Instructions. Your work on this exam will be graded according to two criteria: mathematical correctness and clarity of presentation. In other words, you must know what you are doing (mathematically) and you must also express yourself clearly. In particular, write answers to questions using correct notation and using complete sentences where appropriate. Also, you must supply sufficient detail in your solutions (relevant calculations, written explanations of why you are doing these calculations, etc.). It is not sufficient to just write down an “answer” with no explanation of how you arrived at that answer. As a rule of thumb, the harder that I have to work to interpret what you are trying to say, the less credit you will get. You may use your calculator on this exam but you may not use any books or notes.

1. The Pythagorean Theorem states that if a right triangle has sides of length $a$ and $b$ and hypotenuse of length $c$, then

$$a^2 + b^2 = c^2.$$ 

Give James Garfield’s proof of this theorem. The proof must be written using complete sentences and correct mathematical notation. (It is essential to include a picture with your proof.)

In giving this proof, you make take it as already known that the sum of the angles of any triangle is $180^\circ$ and that the area of a trapezoid with parallel sides of length $b_1$ and $b_2$ and height $h$ is

$$A = \frac{1}{2} (b_1 + b_2) h.$$ 

(You don’t have to prove these two facts.)
2. (a) For the right triangle pictured here, find the six trigonometric ratios of the angle \( \theta \) in terms of \( p, q, \) and \( r \).

\[
\begin{align*}
\sin (\theta) &= \underline{\text{__________}} & \csc (\theta) &= \underline{\text{__________}} \\
\cos (\theta) &= \underline{\text{__________}} & \sec (\theta) &= \underline{\text{__________}} \\
\tan (\theta) &= \underline{\text{__________}} & \cot (\theta) &= \underline{\text{__________}}
\end{align*}
\]

(b) Explain how you would use your calculator to go about finding the angle \( \theta \) assuming that \( p \) and \( r \) are given. (Explain the procedure you would use on your calculator to find \( \theta \).)
3. Explain why

$$\tan(30^\circ) = \frac{\sqrt{3}}{3}.$$ 

Do this by drawing an appropriate picture and using the Pythagorean Theorem. (Be sure to explain what you are doing in words.)
4. For the right triangle pictured here, find angle $A$ and side lengths $a$ and $c$. You must explain your reasoning.
5. In the figure shown below, find the length $x$. You must include all details. (You can round your answer to two decimal places.)
6. Find the six trigonometric function values for the angle, \( \theta \), pictured here. Show all of your work.

\[
\sin (\theta) = \quad \csc (\theta) = \\
\cos (\theta) = \quad \sec (\theta) = \\
tan (\theta) = \quad \cot (\theta) =
\]

Show work here:
7. (Short answer miscellaneous questions)

(a) True or False?: There are certain angles, \( \theta \), for which \( \cos(\theta) \) is not defined. You must explain your answer (not just say true or false).

(b) Suppose that you are told that \( \csc(\theta) \) is a positive number. In which possible quadrants might the terminal side of \( \theta \) lie?

(c) Are the angles 203° and \(-27,833°\) coterminal? You must explain your answer (not just say yes or no!).

(d) \( \tan(75°) = \cot(\underline{\phantom{000}}) \). Explain your answer.

(e) How many different angles, \( \theta \), are there between 0° and 180° such that \( \sin(\theta) = 0.35 \)? You must explain your answer. Your explanation should make reference to the unit circle.
8. Suppose that $\theta$ is between $270^\circ$ and $360^\circ$ and that $\sin(\theta) = -0.79863551$. Find the value of $\theta$ to the nearest degree. You must include a clear explanation of how you go about this.