Instructions. Your work on this exam will be graded according to two criteria: mathematical correctness and clarity of presentation. In other words, you must know what you are doing (mathematically) and you must also express yourself clearly. You may use your calculator on this exam and you may also use the two page summary sheet of trigonometric identities, but you may not use any other books or notes.

Important: In proving trigonometric identities on this exam, you must use one of the two correct methods (what we have been referring to as methods 1 and 2). If you use the incorrect method of proof writing – the method where you begin with writing the identity to be proved \( f(x) = g(x) \) and then reasoning to a true statement \( h(x) = h(x) \), the maximum number of points that you will receive is 4. Also, you must write “=” where it is needed. If there are more than one or two instances of missing “=” in your proofs, then the maximum number of points that you will receive is 4.

1. (Longer problem but you knew it was coming) Use the identity

\[
\cos (\alpha - \beta) = \cos (\alpha) \cos (\beta) + \sin (\alpha) \sin (\beta)
\]

and also the even–odd and cofunction identities to prove all three of the identities

\[
\cos (\alpha + \beta) = \cos (\alpha) \cos (\beta) - \sin (\alpha) \sin (\beta)
\]

\[
\sin (\alpha - \beta) = \sin (\alpha) \cos (\beta) - \cos (\alpha) \sin (\beta)
\]

\[
\sin (\alpha + \beta) = \sin (\alpha) \cos (\beta) + \cos (\alpha) \sin (\beta).
\]

2. Use the sum or difference identity for tangent to show that

\[
\tan \left( \frac{5\pi}{12} \right) = \sqrt{3} + 2.
\]

Include all details!

3. Use the sum identity for cosine:

\[
\cos (\alpha + \beta) = \cos (\alpha) \cos (\beta) - \sin (\alpha) \sin (\beta)
\]

and any other necessary identities (such as perhaps one of the Pythagorean identities) to prove the double angle identity

\[
\cos (2\theta) = 2 \cos^2 (\theta) - 1.
\]

4. Given that \( \sin (\theta) = \frac{1}{5} \) and that the terminal side of \( \theta \) is in Quadrant II, find the exact values of:

   (a) \( \cos (\theta) \)

   (b) \( \sin (2\theta) \)
(c) $\cos (2\theta)$
(d) $\tan (2\theta)$

You must include all details of how you go about this.

5. Prove the identity

$$\frac{\sin (\theta) + \cos (\theta)}{\sec (\theta) + \csc (\theta)} = \frac{\sin (\theta)}{\sec (\theta)}.$$  

6. Prove the identity

$$\frac{\tan (\theta) + \cot (\theta)}{\csc (\theta)} = \sec (\theta).$$