Instructions. Your work on this exam will be graded according to two criteria: mathematical correctness and clarity of presentation. In other words, you must know what you are doing (mathematically) and you must also present your work in a clear and organized way (using correct mathematical notation). You may use your calculator on this exam but you may not use any other books or notes.

This exam contains 7 questions but you only have to do 6 of them – any 6 of your choice. However, you must indicate (by circling in the table below) the number of the question that you do not want me to grade. Even if you work on all 7 questions, I will only grade the 6 that you indicate that you want me to grade.

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1
1. These are short essay questions. You must answer each of them using complete sentences. Two or three sentences should suffice for each.

(a) With my calculator set in radians mode, I type in \( \sin (2.5) \) and I get the answer \( 0.5984721441 \). I then type in \( \sin^{-1} (0.5984721441) \) and get the answer \( 0.6415926536 \) (which is not even close to 2.5). Why is this? Why is it not true that \( \sin^{-1} (\sin (2.5)) = 2.5 \)?

(b) When I use my calculator to try to compute \( \cos^{-1} (\pi) \), I get an error message. Why?

(c) Is there any real number, \( k \), such that \( \tan^{-1} (k) = 2 \)? Explain why or why not.
2. I type $\sin^{-1}(0.23) + \cos^{-1}(0.23)$ into my calculator and the answer that I get is 1. Next, I compute $\sin^{-1}(-0.638) + \cos^{-1}(-0.638)$ and the answer that I get is again 1! Explain why it is always true that

$$\sin^{-1}(x) + \cos^{-1}(x) = 1,$$

no matter what number $x$ (with $-1 \leq x \leq 1$) we use. (Note that what you are actually being asked to do here is to prove the above identity.)
3. Find all solutions of the equation

\[ 4 \tan(x) + 4 = 0 \]

that lie in the interval \([0, 2\pi)\).

Your solution to this problem must contain all of the following:

- algebraic work that shows how you reduce this equation to a solvable form.
- a drawing of the unit circle that indicates how you find the solutions to this equation.
- a graphical check that your solutions are correct. (The “check” graph can be produced on your calculator and then redrawn by hand here.)
- a concluding sentence that is written in the form “The solutions of the equation \(4 \tan(x) + 4 = 0\) that lie in the interval \([0, 2\pi)\) are __________.”
4. Find all solutions of the equation

\[ \sin(2x) + \sqrt{3}\cos(x) = 0 \]

that lie in the interval \([0^\circ, 360^\circ]\).

Your solution to this problem must contain all of the following:

- algebraic work that shows how you reduce this equation to a solvable form.
- a drawing of the unit circle that indicates how you find the solutions to this equation.
- a graphical check that your solutions are correct. (The “check” graph can be produced on your calculator and then redrawn by hand here.)
- a concluding sentence that is written in the form “The solutions of the equation \( \sin(2x) + \sqrt{3}\cos(x) = 0 \) that lie in the interval \([0^\circ, 360^\circ]\) are ________.”
5. Prove the **Law of Sines**,

\[
\frac{a}{\sin(A)} = \frac{b}{\sin(B)},
\]

in the cases illustrated in each of the two figures given below.
6. Find the angles \((B \text{ and } C)\) and the side length \(c\) of all triangles that satisfy

\[
\begin{array}{|c|c|}
\hline
\text{A} & \text{a} = 24 \\
\hline
\text{B} & \text{b} = 34 \\
\hline
\text{C} & \text{c} = ? \\
\hline
\end{array}
\]

Your solution must include all details of how you go about this (whether you are using the Law of Sines, Law of Cosines, etc.). You should also indicate whether there are no solutions, one solution, or two solutions.
7. Find the angles ($A$, $B$, and $C$) of all triangles that satisfy

<table>
<thead>
<tr>
<th>$A =$?</th>
<th>$a = 26$</th>
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<tbody>
<tr>
<td>$B =$?</td>
<td>$b = 21$</td>
</tr>
<tr>
<td>$C =$?</td>
<td>$c = 19$</td>
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</tbody>
</table>

Your solution must include all details of how you go about this (whether you are using the Law of Sines, Law of Cosines, etc.). You should also indicate whether there are no solutions, one solution, or two solutions.