Limits of Difference Quotients – Answers to Exercises

January 22, 2001

Exercise Set 1.1

1. The slope of \( f(x) = x^2 \) at \( x_0 = -3 \) is \(-6\).

3. The slope of \( f(x) = \cos x \) at \( x_0 = 0 \) is 0.

5. The equation of the tangent line to the graph of \( f(x) = x^2 \) at \( x_0 = -3 \) is \( y = -6(x+3) + 9 \). The equation of the tangent line to the graph of \( f(x) = \cos x \) at \( x_0 = 0 \) is \( y = 1 \).

Exercise Set 3.1

1. Take \( a = 0.000000014 \) and \( b = 0.000000001 \). Then \( a/b = 14 \).

3. We want both \( a \) and \( b \) to be positive numbers that are both less than some given small positive number \( \epsilon \) and such that \( a/b = 5 \). To achieve this, let \( a \) be any positive number less than \( \epsilon \) and then let \( b = \frac{1}{5} a \). We then have

\[ 0 < b < a < \epsilon \]

and \( a/b = 5 \).

4. (a) \( \lim_{x \to 3} \frac{14x-42}{x-3} = 14 \).

(b) \( \lim_{x \to 0} \frac{2x^2+4x+2}{x^2+4x+2} = 3 \).

(c) \( \lim_{x \to -2} \frac{2x^2-9x-5}{4x^2-20x} = \frac{11}{30} \).
(e) \( \lim_{x \to 1} \frac{x^2 - 1}{x - 1} = 3. \)

(f) \( \lim_{x \to 4} \frac{x^2 - 2}{x - 4} = \frac{1}{4}. \)

6. The linear function that best approximates \( f(x) = \sqrt{x} \) for values of \( x \) near \( x_0 = 4 \) is \( L(x) = \frac{1}{4} (x - 4) + 2. \)

9. The tangent line to the graph of \( f(x) = \frac{1}{x} \) at the point \((1, 1)\) has equation \( y = - (x - 1) + 1 \) and the tangent line to the graph of \( g(x) = \frac{1}{x^2} \) at the point \((1, 1)\) has equation \( y = -2 (x - 1) + 1. \) This means that the graph of \( g \) is “twice as steep” as the graph of \( f \) at the point \((1, 1)\) (which is on both graphs). This fact is supported by the graphs of \( f \) and \( g \) shown below.