Instructions. Your work on this exam will be graded according to two criteria: mathematical correctness and clarity of presentation. In other words, you must know what you are doing (mathematically) and you must also express yourself clearly. In particular, write answers to questions using correct notation and using complete sentences where appropriate. Also, you must supply sufficient detail in your solutions (relevant calculations, written explanations of why you are doing these calculations, etc.). It is not sufficient to just write down an “answer” with no explanation of how you arrived at that answer. As a rule of thumb, the harder that I have to work to interpret what you are trying to say, the less credit you will get. You may use your calculator on this exam but you may not use any books or notes.

1. Write the point–slope form equation of the line

\[ y = 10x - 7 \]

that emphasizes the fact that the point \((-3, -37)\) is on this line.

Solution: The equation of this line can be written in the point–slope form

\[ y - (-37) = 10(x - (-3)). \]

(a) Graph the function

\[ f(x) = x^2 - 5x - 2. \]

(b) Based on the graph that you have drawn, determine the intervals on which the function \(f\) is increasing and the intervals on which \(f\) is decreasing. You must write your answers in complete sentences using the correct terminology. For example, “\(f\) is increasing on the interval \((-\infty, \_\_\_\_)\)”.

Answers: \(f\) is decreasing on the interval \((-\infty, 2.5)\) and \(f\) is increasing on the interval \((2.5, \infty)\).
(c) Fill in the blanks:

1. \( f \) has no relative maximums.
2. \( f \) has a relative minimum value of \(-8.25\) occurring at 2.5.

2. Fill in the blanks:

If a function \( f \) is decreasing on the interval \((1, 2)\) and increasing on the interval \((2, 3)\) and if \( f(1) = 3, f(2) = -9, \) and \( f(3) = -2, \) then \( f \) has a relative minimum value of \(-9\) occurring at 2.

3. A graph of the function

\[ f(x) = \sqrt{x^2 + 16} \]

is shown below.

Since \( f(-3) = 5 \), we can see that the point \((-3, 5)\) is on the graph of \( f \).

(a) Fill in the following table which gives the slopes of certain secant lines. (Numbers can be rounded to five decimal places.) You will need your calculator to do these calculations, but you must show the steps of one sample calculation.

<table>
<thead>
<tr>
<th>point ((x, f(x)))</th>
<th>slope of the secant line joining ((-3, 5)) to ((x, f(x)))</th>
</tr>
</thead>
<tbody>
<tr>
<td>((-3.1, \sqrt{(-3.1)^2 + 16}))</td>
<td>-0.60632</td>
</tr>
<tr>
<td>((-2.9, \sqrt{(-2.9)^2 + 16}))</td>
<td>-0.59352</td>
</tr>
<tr>
<td>((-3.01, \text{etc.}))</td>
<td>-0.60064</td>
</tr>
<tr>
<td>((-2.99, \text{etc.}))</td>
<td>-0.59936</td>
</tr>
<tr>
<td>((-3.001, \text{etc.}))</td>
<td>-0.60006</td>
</tr>
<tr>
<td>((-2.999, \text{etc.}))</td>
<td>-0.59994</td>
</tr>
<tr>
<td>((-3.000453, \text{etc.}))</td>
<td>-0.60003</td>
</tr>
</tbody>
</table>
Show a sample calculation here: The slope of the secant line joining the point 
\((-3, 5)\) to the point \(\left(-3.1, \sqrt{(-3.1)^2 + 16}\right)\) is

\[
\frac{\sqrt{(-3.1)^2 + 16} - 5}{(-3.1) - (-3)} \approx -0.60632.
\]

(b) Based on my calculations (in part a above), I estimate that the slope of the
tangent line to the graph of \(f(x) = \sqrt{x^2 + 16}\) at the point \((-3, 5)\) is \(-0.6\).

(c) Assuming that the slope that I estimated (in part b above) is correct, the equation
of the tangent line to the graph of \(f(x) = \sqrt{x^2 + 16}\) at the point \((-3, 5)\) is

\[y - 5 = -0.6(x + 3)\]
or
\[y = -0.6x + 3.2\]

(d) Graph \(f\) together with the tangent line that you found in part c. (You can use
your calculator to do this.) Show the graphs below. Does it appear that you have
found the correct tangent line?

4. In problem 4, you obtained a numerical estimate of the slope of the tangent line to
the graph of \(f(x) = \sqrt{x^2 + 16}\) at the point on the graph corresponding to \(x_0 = -3\).
The point of this problem is to find the exact value of this slope (algebraically) by
computing \(f'(-3)\). In doing this, you must use one of the two (equivalent) definitions
of the derivative, either

\[f'(x_0) = \lim_{x \to x_0} \frac{f(x) - f(x_0)}{x - x_0}\]

or

\[f'(x_0) = \lim_{h \to 0} \frac{f(x_0 + h) - f(x_0)}{h}.
\]
and you must include all of the relevant algebraic details of your calculations.

**Solution:**

\[
f'(-3) = \lim_{x \to -3} \frac{f(x) - f(-3)}{x - (-3)}
\]

\[
= \lim_{x \to -3} \frac{\sqrt{x^2 + 16} - 5}{x + 3} \cdot \frac{\sqrt{x^2 + 16} + 5}{\sqrt{x^2 + 16} + 5}
\]

\[
= \lim_{x \to -3} \frac{x^2 + 16 - 25}{(x + 3)(\sqrt{x^2 + 16} + 5)}
\]

\[
= \lim_{x \to -3} \frac{x^2 - 9}{(x + 3)(\sqrt{x^2 + 16} + 5)}
\]

\[
= \lim_{x \to -3} \frac{(x + 3)(x - 3)}{(x + 3)(\sqrt{x^2 + 16} + 5)}
\]

\[
= \lim_{x \to -3} \frac{x - 3}{\sqrt{x^2 + 16} + 5}
\]

\[
= \frac{-3 - 3}{\sqrt{(-3)^2 + 16} + 5}
\]

\[
= \frac{-6}{10}
\]

\[
= -0.6.
\]

5. One of the limit problems below is what we would refer to as an “easy” limit problem and one is what we would refer to as a “hard” limit problem (because it is a 0/0 indeterminate form limit problem). Evaluate both limits. State which one is the “easy” one and which one is the “hard” one. Show all details of your work in evaluating each limit.

(a)

\[
\lim_{x \to 2} \frac{x^2 - 2x}{x^2 + x - 6} = ?
\]

**Show work here:** This is a “hard” limit problem, but we can convert it to an easy one by realizing that for all \(x \neq 2\), we have

\[
\frac{x^2 - 2x}{x^2 + x - 6} = \frac{x(x - 2)}{(x - 2)(x + 3)} = \frac{x}{x + 3}.
\]

Thus

\[
\lim_{x \to 2} \frac{x^2 - 2x}{x^2 + x - 6} = \lim_{x \to 2} \frac{x}{x + 3} = \frac{2}{2 + 3} = \frac{2}{5}.
\]

(b)

\[
\lim_{x \to 3} \frac{x^2 - 2x}{x^2 + x - 6} = ?
\]
**Show work here:** This is an “easy” limit problem.

\[
\lim_{x \to 3} \frac{x^2 - 2x}{x^2 + x - 6} = \frac{(3)^2 - 2(3)}{(3)^2 + 3 - 6} = \frac{3}{6} = \frac{1}{2}
\]

6. Find the derivative function, \( f' \), of the function \( f(x) = x^2 - 5x - 2 \). In doing this, you must use one of the two (equivalent) definitions of the derivative, either

\[
f'(x_0) = \lim_{x \to x_0} \frac{f(x) - f(x_0)}{x - x_0}
\]

or

\[
f'(x_0) = \lim_{h \to 0} \frac{f(x_0 + h) - f(x_0)}{h},
\]

and you must include all of the relevant algebraic details of your calculations. The answer that you should obtain is \( f'(x) = 2x - 5 \).

**Solution:**

\[
f'(x) = \lim_{h \to 0} \frac{f(x + h) - f(x)}{h}
\]

\[
= \lim_{h \to 0} \frac{(x + h)^2 - 5(x + h) - 2 - (x^2 - 5x - 2)}{h}
\]

\[
= \lim_{h \to 0} \frac{x^2 + 2xh + h^2 - 5x - 5h - 2 - x^2 + 5x + 2}{h}
\]

\[
= \lim_{h \to 0} \frac{2xh + h^2 - 5h}{h}
\]

\[
= \lim_{h \to 0} (2x + h - 5)
\]

\[
= 2x + 0 - 5
\]

\[
= 2x - 5.
\]