What Does The Derivative of $f$ Tell Us About $f$?

**Theorem** Suppose that $f$ is a function that is differentiable at all points in some interval $I$.

1. If $f'(x) > 0$ for all $x \in I$, then $f$ is increasing on $I$.
2. If $f'(x) < 0$ for all $x \in I$, then $f$ is decreasing on $I$. 
Example  The graph of the derivative, $f'$, of a function $f$ is shown below. What does this tell us about $f$? Suppose that it is also known that $f(0) = 0$. Make a rough sketch of the graph of $f$ in this case.

Graph of $f'$
What Does The Second Derivative of $f$ Tell Us About $f$?

**Theorem**  Suppose that $f$ is a function that is twice differentiable at all points in some interval $I$.

1. If $f''(x) > 0$ for all $x \in I$, then $f$ is concave up on $I$.
2. If $f''(x) < 0$ for all $x \in I$, then $f$ is concave down on $I$. 
**Example** Sketch a possible graph of a function, \( f \), that satisfies all of the following conditions:

- \( f'(x) > 0 \) for all \( x \in (-\infty, 1) \) and \( f'(x) < 0 \) for all \( x \in (1, \infty) \).
- \( f''(x) > 0 \) for all \( x \in (-\infty, -2) \), and \( f''(x) < 0 \) for all \( x \in (-2, 2) \), and \( f''(x) > 0 \) for all \( x \in (2, \infty) \).
- \( \lim_{x \to -\infty} f(x) = -2 \) and \( \lim_{x \to \infty} f(x) = 0 \).
Antiderivatives

**Definition** Suppose that $f$ is a function whose domain includes some interval $I$. A function, $F$, is called an antiderivative of $f$ on $I$ if $F'(x) = f(x)$ for all $x \in I$.

**Example** The function $F(x) = x^2$ is an antiderivative of the function $f(x) = 2x$ on the interval $(-\infty, \infty)$ because (as we saw in an earlier example) $F'(x) = f(x)$ for all $x \in (-\infty, \infty)$.

However, the function $F(x) = x^2 + 6$ is also an antiderivative of the function $f(x) = 2x$ on the interval $(-\infty, \infty)$. In fact, if $C$ is any constant, then the function $F(x) = x^2 + C$ is an antiderivative of the function $f(x) = 2x$ on the interval $(-\infty, \infty)$. This is why we use the word “an” (rather than “the”) when referring to antiderivatives. When a
function $f$ has an antiderivative on an interval $I$, then $f$ always, in fact, has infinitely many antiderivatives on $I$. 
Example Let $f$ be the function with domain $[0, 5]$ whose graph is shown below and let $F$ be an antiderivative of $f$.

1. On which intervals is $F$ increasing and on which intervals is $F$ decreasing?
2. On which intervals is $F$ concave up and on which intervals is $F$ concave down?
3. At which values of $x$ does $F$ have an inflection point?
4. Suppose that $F(0) = 1$ and make a rough sketch of the graph of $F$.
5. How many antiderivatives does $f$
have?