These notes closely follow the presentation of the material given in James Stewart’s textbook Calculus, Concepts and Contexts (2nd edition). These notes are intended primarily for in-class presentation and should not be regarded as a substitute for thoroughly reading the textbook itself and working through the exercises therein.

The Product Rule of Differentiation

If $f$ and $g$ are differentiable functions, then

$$(f \cdot g)' = f \cdot g' + f' \cdot g.$$

**Example** Find the derivative function of the function $F(x) = x \cdot e^x$. 
Example  A telephone company wants to estimate the number of new residential phone lines that it will need to install during the upcoming month. At the beginning of January, the company had 100,000 subscribers, each of whom had 1.2 phone lines on average. The company estimated that its subscribepship was increasing at the rate of 1000 customers monthly. By polling its existing subscribers, the company found that each intended to install an average of 0.01 new phone lines by the end of January.

Estimate the number of new lines the company will have to install in January by calculating the rate of increase in lines at the beginning of the month.

Solution  Let $s(t)$ be the number of subscribers and let $n(t)$ be the number of phone lines per subscriber at time $t$, where $t$ is measured in months and $t = 0$ is the beginning of January.
The Quotient Rule of Differentiation

If \( f \) and \( g \) are differentiable functions, then
\[
\left( \frac{f}{g} \right)' = \frac{g \cdot f' - f \cdot g'}{g^2}.
\]
This is true at all points, \( x \), at which \( g(x) \neq 0 \).

**Example**  Find the derivative of the function
\[
F(x) = \frac{x + 1}{2 - x}.
\]
Example Use the Quotient Rule to show that the derivative function of the function

$$f(x) = \frac{x}{x^2 - 1}$$

is the function

$$f'(x) = -\frac{x^2 + 1}{(x^2 - 1)^2}.$$ 

The graphs of each of these functions are shown below. Convince yourself that the two graphs make sense in relation to each other.

Graph of $f(x) = \frac{x}{x^2 - 1}$

Graph of $f'(x) = -\frac{x^2 + 1}{(x^2 - 1)^2}$