Instructions. Your work on this exam will be graded according to two criteria: **mathematical correctness** and **clarity of presentation**. In other words, you must know what you are doing (mathematically) and you must also express yourself clearly. In particular, write answers to questions using **complete sentences** where appropriate. Also, you must supply sufficient detail in your solutions (relevant calculations, written explanations of why you are doing these calculations, etc.). It is not sufficient to just write down an “answer” with no explanation of how you arrived at that answer. As a rule of thumb, the harder that I have to work to interpret what you are trying to say, the less credit you will get. You may use your calculator but you may not use any books or notes.

1. Circle **True** or **False** for each of the following statements.

   (a) If a function \( f \) is continuous at a number \( a \), then \( \lim_{x \to a} f(x) = f(a) \). (**True**)
   
   (b) If \( \lim_{x \to a} f(x) \) exists, then the function \( f \) must be continuous at the number \( a \). (**False**)
   
   (c) If \( \lim_{x \to a} f(x) \) and \( \lim_{x \to a} g(x) \) both exist, then
   \[
   \lim_{x \to a} (f(x) - g(x)) = \lim_{x \to a} f(x) - \lim_{x \to a} g(x) .
   \]
   (**True**)

   (d) If \( \lim_{x \to a} f(x) = \infty \) and \( \lim_{x \to a} g(x) = \infty \), then it must be true that
   \[
   \lim_{x \to a} \frac{f(x)}{g(x)} = \infty.
   \]
   (**False**)

   (e) If \( \lim_{x \to a} f(x) = \infty \) and \( \lim_{x \to a} g(x) = \infty \), then it must be true that
   \[
   \lim_{x \to a} \frac{f(x)}{g(x)} = 1 .
   \]
   (**False**)

2. Consider the limit problem:

   \[
   \lim_{x \to 2} \frac{1}{x} - \frac{1}{x^2} .
   \]

   (a) Guess an approximate answer to the above problem by completing the following table:

<table>
<thead>
<tr>
<th>( x )</th>
<th>( \frac{1}{x} - \frac{1}{x^2} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.1</td>
<td>-0.2381</td>
</tr>
<tr>
<td>1.9</td>
<td>-0.2632</td>
</tr>
<tr>
<td>2.05</td>
<td>-0.2439</td>
</tr>
<tr>
<td>1.95</td>
<td>-0.2564</td>
</tr>
<tr>
<td>2.0023</td>
<td>-0.2497</td>
</tr>
<tr>
<td>1.0077</td>
<td>-0.4962</td>
</tr>
</tbody>
</table>
**Guess:** Based on numerical calculations, the value of the above limit is approximately equal to ___-0.25___________. (Note: The last choice of 

\( x = 1.0077 \) is not really relevant in guessing the limit since it is not very close to 2.)

(b) Use algebra to find the exact value of the limit in question. (Show your solution in detail.)

Since for all \( x \neq 2 \) we have

\[
\frac{1}{x} - \frac{1}{2} = \frac{2-x}{2x} = \frac{2-x}{x-2} \cdot \frac{1}{x-2} = \frac{-1}{2x},
\]

then

\[
\lim_{x \to 2} \frac{1}{x} - \frac{1}{2} = \lim_{x \to 2} \frac{-1}{2x} = \frac{-1}{2 \cdot 2} = -\frac{1}{4}.
\]

3. For the function, \( f \), whose graph is given, state the value of the given quantity if it exists. If it does not exist, then explain why.

\[\text{graph of } y = f(x)\]

(a) \( \lim_{x \to -2} f(x) = 0 \)

(b) \( f(-2) \) is not defined. There is no point on the graph corresponding to \( x = -2 \).

(c) \( \lim_{x \to 1} f(x) \) does not exist because \( \lim_{x \to 1^-} f(x) = 1 \) and \( \lim_{x \to 1^+} f(x) = -2 \).

(d) \( f(1) = 0 \).
4. Use either the definition

\[ m = \lim_{x \to a} \frac{f(x) - f(a)}{x - a} \]

or the definition

\[ m = \lim_{h \to 0} \frac{f(a + h) - f(a)}{h} \]

(whichever definition you prefer) to show that the parabola

\[ y = 2x^2 + 3x \]

has tangent line with slope \( m = -5 \) at the point \( P(-2, 2) \).

(Include all relevant details so that it is clear to the reader exactly how one or the other of the above definitions is being used.)

For all \( h \neq 0 \) we have

\[
\frac{f(a + h) - f(a)}{h} = \frac{f(-2 + h) - f(-2)}{h} = \frac{2(-2 + h)^2 + 3(-2 + h) - 2}{h} = \frac{2(4 - 4h + h^2) + 3(-2 + h) - 2}{h} = \frac{8 - 8h + 2h^2 - 6 + 3h - 2}{h} = \frac{-5h + 2h^2}{h} = -5 + 2h.
\]

Thus the slope of the tangent line to the curve \( y = 2x^2 + 3x \) at the point \( P(-2, 2) \) is

\[ m = \lim_{h \to 0} \frac{f(-2 + h) - f(-2)}{h} = \lim_{h \to 0} (-5 + 2h) = -5 + 2 \cdot 0 = -5. \]

5. Match each function in (a)–(e) with the graphs in (1)–(5). The correct matchings are (1,d), (2,b), (3,c), (4,a), and (5,e).
\[
\begin{align*}
\text{a)} & \quad y = \frac{x + 1}{x^2 + 5x + 6} \\
\text{b)} & \quad y = \frac{x + 1}{x^2 - x - 2} \\
\text{c)} & \quad y = \frac{6}{x + 1} \\
\text{d)} & \quad y = \frac{x^2 + 2x + 1}{x^2 - x - 2} \\
\text{e)} & \quad y = \frac{x^2 + 5x + 6}{x + 1}
\end{align*}
\]