Homework Assignment 16

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1. Find the magnitude and the direction angles of the vector $\vec{v} = (1, -6, 2)$.

2. Find the vector, $\vec{v}'$, which has magnitude 4 and direction angles $\alpha = \pi/2$, $\beta = \pi/2$, and $\gamma = 0$.

3. Find three different representatives of the vector $\vec{v} = (4, -2, 0)$. (Recall that a representative of $\vec{v}$ is a directed line segment with same magnitude and direction as $\vec{v}$. See Section 1.1 of my notes.) What is the standard representative of $\vec{v}'$?

4. Let $\vec{v} = (9, -3, -6)$. Find the representative of $\vec{v}$ which has initial point at $P_0 (0, -12, 4)$. Find the representative of $\vec{v}$ which has terminal point at $P_1 (4, 0, -10)$. Find the standard representative of $\vec{v}$.

5. Let $\vec{u} = (2, 6, 0)$, $\vec{v} = (1, 1, 4)$, and $t = -3$. Compute $\vec{u} + \vec{v}$ and $t \vec{v}$.

6. Let $\vec{u} = (5, 3/2, 4/5)$ and $\vec{v} = (6, -6, 1/5)$. Compute $2 \vec{u} - 5 \vec{v}$.

7. Let $\vec{v} = (9, 2, -1/2)$. Find a vector whose magnitude is four times the magnitude of $\vec{v}$ and which points in the opposite direction of $\vec{v}$. Also, find a vector whose magnitude is one tenth the magnitude of $\vec{v}$ and which points in the same direction as $\vec{v}$.

8. Is $\vec{u} = (1/4, 1/2, -1/4)$ a unit vector?

9. Find a unit vector which points in the direction of $\vec{v} = (9, 2, -1/2)$.

10. Given that $\vec{u}$ and $\vec{v}$ are vectors with $|\vec{u}| = 2$, $\vec{u} \cdot \vec{v} = 3$, and the angle between $\vec{u}$ and $\vec{v}$ is $\theta = \pi/4$, find $|\vec{v}|$. 

1
11. Use the dot product to show that the vectors $\mathbf{u}' = 5 \mathbf{i} - 2 \mathbf{j} + \mathbf{k}$ and $\mathbf{v}' = 2 \mathbf{i} - 3 \mathbf{j} - 16 \mathbf{k}$ are perpendicular.

12. Let $\mathbf{u}' = 4 \mathbf{i} + \frac{1}{2} \mathbf{j}$ and $\mathbf{v}' = 2 \mathbf{i} - \frac{1}{2} \mathbf{k}$. Find the sine of the angle, $\theta$, between $\mathbf{u}'$ and $\mathbf{v}'$. (Hint: You should easily be able to obtain $\cos \theta$ by using the dot product.) Also find $|\mathbf{u}'|$, $|\mathbf{v}'|$, and $|\mathbf{u}' \times \mathbf{v}'|$. Then, verify that $|\mathbf{u}' \times \mathbf{v}'| = |\mathbf{u}'||\mathbf{v}'| \sin \theta$. Do all of this without using a calculator or computer.

13. Use the cross product to verify that the vectors $\mathbf{u}' = 7 \mathbf{i} - 3 \mathbf{j} - \frac{4}{5} \mathbf{k}$ and $\mathbf{v}' = -21 \mathbf{i} + 9 \mathbf{j} + 4 \mathbf{k}$ are parallel. Do you see an easier way to see that $\mathbf{u}'$ and $\mathbf{v}'$ are parallel?

14. Find parametric equations for the line, $L_1$, which contains the points $P(0, -6, 4)$ and $Q(1, 1, -6)$. Once you have done this, verify that the point $P_1(-3, -27, 34)$ is on $L_1$ but the point $P_2(-1, -10, 14)$ is not on $L_1$.

15. Find parametric equations for the line, $L_2$, which is parallel to the line, $L_1$, of the previous problem, and which contains the point $P_2(-1, -10, 14)$.

16. Find an equation for the plane, $\Pi_1$, which contains the points $P(0, 0, -7)$, $Q(1, 2, -4)$, and $R(-1, 0, 1)$. After you have done this, verify that the point $P_1(2, 1, -35/2)$ is on $\Pi_1$ but that the point $P_2(5, -3/2, 3/2)$ is not on $\Pi_1$.

17. Find an equation for a plane, $\Pi_2$, which is parallel to the plane, $\Pi_1$, of the previous problem and which contains the point $P_2(5, -3/2, 3/2)$.

18. Let $L$ be the line with parametric equations

$$
\begin{align*}
  x &= 6 - t \\
  y &= 4 - 2t \\
  z &= 3
\end{align*}
$$

and let $P$ be the point $P(0, 0, 4)$. Find the distance from $P$ to $L$.

19. Find the distance between the point $P(0, 0, 4)$ and the plane $\Pi : 3x + 2z = 0$. 

2
20. Let \( L_1 \) be the line with parametric equations
\[
\begin{align*}
x &= 6 - t \\
y &= 4 - 2t \\
z &= 2 + t
\end{align*}
\]
and let \( L_2 \) be the line with parametric equations
\[
\begin{align*}
x &= 5 - 5t \\
y &= 2 - 10t \\
z &= 3 + 5t
\end{align*}
\]
Show that \( L_1 \) and \( L_2 \) are the same line.

21. Let \( L_1 \) be the line with parametric equations
\[
\begin{align*}
x &= 6 - t \\
y &= 4 - 2t \\
z &= 2 + t
\end{align*}
\]
and let \( L_2 \) be the line with parametric equations
\[
\begin{align*}
x &= 4 + 3t \\
y &= 4 + 6t \\
z &= 4 - 3t
\end{align*}
\]
Show that \( L_1 \) and \( L_2 \) are parallel (but not the same line) and find the distance between \( L_1 \) and \( L_2 \).

22. Let \( L_1 \) be the line with parametric equations
\[
\begin{align*}
x &= 6 - t \\
y &= 4 - 2t \\
z &= 2 + t
\end{align*}
\]
and let \( L_2 \) be the line with parametric equations
\[
\begin{align*}
x &= 1 + 2t \\
y &= 4 - 6t \\
z &= 4 + t
\end{align*}
\]
Show that \( L_1 \) and \( L_2 \) intersect at a single point and find this point.
23. Let $L_1$ be the line with parametric equations

\[ \begin{align*}
  x &= 6 - t \\
  y &= 4 - 2t \\
  z &= 2 + t
\end{align*} \]

and let $L_2$ be the line with parametric equations

\[ \begin{align*}
  x &= 4 + 2t \\
  y &= -6t \\
  z &= 4
\end{align*} \]

Show that $L_1$ and $L_2$ are skew and find the distance between $L_1$ and $L_2$.

24. Find the area of the triangle with vertices at the points $P(0, -4, 0)$, $Q(1, 2, -10)$, and $R(0, 2, 7)$.