Instructions. This exam contains five indefinite integrals (with answers given). On each problem you must show a) how to evaluate the integral (using the strategies we have been studying such as basic substitution, integration by parts, trigonometric substitution, etc.) and b) how to check that the answer you get is correct (by differentiation). An example of how your work should appear is given here:

Example:

\[ \int x \sec^2 (x^2) \, dx = \frac{1}{2} \tan (x^2) + C. \]

a) To evaluate the above integral, we use a basic substitution: Let \( u = x^2 \). Then \( du = 2x \, dx \) and we have

\[ \int x \sec^2 (x^2) \, dx = \frac{1}{2} \int \sec^2 (u) \, du = \frac{1}{2} \tan (u) + C = \frac{1}{2} \tan (x^2) + C. \]

b) To check that this result is correct, we observe that

\[ \frac{d}{dx} \left( \frac{1}{2} \tan (x^2) \right) = \frac{1}{2} \sec^2 (x^2) \cdot \frac{d}{dx} (x^2) = \frac{1}{2} \sec^2 (x^2) \cdot 2x = x \sec^2 (x^2). \]

Some of the following trigonometric identities will be needed for certain problems:

\[
\begin{array}{|c|c|}
\hline
\sin^2 (\theta) + \cos^2 (\theta) = 1 & \sin (\theta) \cos (\theta) = \frac{1}{2} \sin (2\theta) \\
\tan^2 (\theta) + 1 = \sec^2 (\theta) & \cos^2 (\theta) = \frac{1}{2} (1 + \cos (2\theta)) \\
1 + \cot^2 (\theta) = \csc^2 (\theta) & \sin^2 (\theta) = \frac{1}{2} (1 - \cos (2\theta)) \\
\hline
\end{array}
\]

1. \[ \int \arctan (4x) \, dx = x \arctan (4x) - \frac{1}{8} \ln (16x^2 + 1) + C. \]

2. \[ \int \sqrt{25 - x^2} \, dx = \frac{1}{2} \left( x \sqrt{25 - x^2} + 25 \arcsin \left( \frac{1}{5} x \right) \right) + C \]

3. \[ \int x^3 \sqrt{2x^4 - 1} \, dx = \frac{1}{12} (2x^4 - 1)^{3/2} + C. \]

4. \[ \int \frac{x^4}{x^2 - 1} \, dx = x + \frac{1}{3} x^3 + \frac{1}{2} \ln (x - 1) - \frac{1}{2} \ln (x + 1) + C. \]

5. \[ \int \sin^3 (x) \cos^3 (x) \, dx = ? \]

(Note that the answer is not give on this one. Just show how to compute the integral. You do not have to show the check of your answer on this one.)