1. Suppose that we have a series $\sum_{n=1}^{\infty} a_n$ and suppose that we have discovered that $\lim_{n \to \infty} a_n = 0$. Does this allow us to conclude anything about the convergence or divergence of the series $\sum_{n=1}^{\infty} a_n$? (Circle the correct answer.)

(a) Yes, we can conclude that the series $\sum_{n=1}^{\infty} a_n$ converges.
(b) Yes, we can conclude that the series $\sum_{n=1}^{\infty} a_n$ diverges.
(c) No, there is nothing that can be concluded about the convergence or divergence of the series $\sum_{n=1}^{\infty} a_n$.

2. Consider the series

$$\sum_{n=1}^{\infty} \frac{n}{5n^2 + 2}.$$ 

Determine whether this series converges or diverges and use one of the convergence/divergence tests that we have studied to justify your answer. (Write in complete sentences and include details of calculations, correct notation, etc. Do not use the word “it” anywhere in your writing.)

**Answer:** This series diverges. We can use either the Integral Comparison Test (ICT), the Standard Comparison Test (SCT), or the Limit Comparison Test (LCT) to prove this. We will use the LCT.

Let $a_n = n / (5n^2 + 2)$ and $b_n = 1/n$. Note that $a_n > 0$ and $b_n > 0$ for all $n \geq 1$. Also

$$\lim_{n \to \infty} \frac{a_n}{b_n} = \lim_{n \to \infty} \frac{n^2 / (5n^2 + 2)}{1} = \frac{1}{5}.$$ 

Since the above limit exists and is not equal to 0 (or to $\infty$), then the LCT tells us that the two series

$$\sum_{n=1}^{\infty} \frac{n}{5n^2 + 2}$$ 

and

$$\sum_{n=1}^{\infty} \frac{1}{n}$$
must either both converge or both diverge. Since the harmonic series,
\[ \sum_{n=1}^{\infty} \frac{1}{n} \], is well-known to us to be divergent, we conclude that the series
\[ \sum_{n=1}^{\infty} \frac{n}{5n^2+2} \] is also divergent.