Find a power series representation for the function 

\[ f(x) = \frac{x^3}{(1 - x)^2} \]

and determine the radius of convergence of this power series.

*Hint:* Begin with the fact that 

\[ \frac{1}{1 - x} = 1 + x + x^2 + x^3 + \cdots \]

for all \( x \) such that \( |x| < 1 \) and then use the fact that 

\[ \frac{d}{dx} \left( \frac{1}{1 - x} \right) = \frac{1}{(1 - x)^2} \]

You must show all of your work in detail (well-written).

**Solution:** Using the facts given above (the hints) we have 

\[ \frac{1}{(1 - x)^2} = 1 + 2x + 3x^2 + 4x^3 + \cdots \]

for all \( x \) such that \( |x| < 1 \). Also since 

\[ \frac{x^3}{(1 - x)^2} = x^3 \cdot \frac{1}{(1 - x)^2} \]

we have 

\[ \frac{x^3}{(1 - x)^2} = x^3 + 2x^4 + 3x^5 + 4x^6 + \cdots \]

for all \( x \) such that \( |x| < 1 \).

Since the above power series was obtained by term-by-term differentiation of a power series whose radius of convergence is 1 (and then by multiplication by \( x^3 \)), we know that the radius of convergence of the new power series is also \( R = 1 \).