Instructions. This exam contains five indefinite integrals (with answers given). On each problem you must show a) how to evaluate the integral (using the strategies we have been studying such as basic substitution, integration by parts, trigonometric substitution, etc.) and b) how to check that the answer you get is correct (by differentiation). An example of how your work should appear is given here:

Example:

\[ \int x \sec^2(x^2) \, dx = \frac{1}{2} \tan(x^2) + C. \]

a) To evaluate the above integral, we use a basic substitution: Let \( u = x^2 \). Then \( du = 2x \, dx \) and we have
\[ \int x \sec^2(x^2) \, dx = \frac{1}{2} \int \sec^2(u) \, du = \frac{1}{2} \tan(u) + C = \frac{1}{2} \tan(x^2) + C. \]

b) To check that this result is correct, we observe that
\[ \frac{d}{dx} \left( \frac{1}{2} \tan(x^2) \right) = \frac{1}{2} \sec^2(x^2) \cdot \frac{d}{dx} (x^2) = \frac{1}{2} \sec^2(x^2) \cdot 2x = x \sec^2(x^2). \]

Some of the following trigonometric identities will be needed for certain problems:

<table>
<thead>
<tr>
<th>Trigonometric Identity</th>
<th>Equivalent Expression</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \sin^2(\theta) + \cos^2(\theta) = 1 )</td>
<td>( \sin(\theta) \cos(\theta) = \frac{1}{2} \sin(2\theta) )</td>
</tr>
<tr>
<td>( \tan^2(\theta) + 1 = \sec^2(\theta) )</td>
<td>( \cos^2(\theta) = \frac{1}{2} (1 + \cos(2\theta)) )</td>
</tr>
<tr>
<td>( 1 + \cot^2(\theta) = \csc^2(\theta) )</td>
<td>( \sin^2(\theta) = \frac{1}{2} (1 - \cos(2\theta)) )</td>
</tr>
</tbody>
</table>

1. \[ \int \frac{\sin(\sqrt{x})}{\sqrt{x}} \, dx = -2 \cos(\sqrt{x}) + C. \]

2. \[ \int \arctan(4x) \, dx = x \arctan(4x) - \frac{1}{8} \ln(1 + 16x^2) + C. \]

3. \[ \int \sin^5(x) \cos^3(x) \, dx = \frac{1}{6} \sin^6(x) - \frac{1}{8} \sin^8(x) + C \]

or
\[ \int \sin^5(x) \cos^3(x) \, dx = -\frac{1}{4} \cos^4(x) + \frac{1}{3} \cos^6(x) - \frac{1}{8} \cos^8(x) + C. \]

(Both of the above formulas are correct. You only need to verify one of them. Also, you do not need to show the check on this one.)

4. \[ \int \frac{\sqrt{4 - x^2}}{x^2} \, dx = -\frac{\sqrt{4 - x^2}}{x} - \arcsin\left( \frac{1}{2} x \right) + C. \]

5.
\[
\int \frac{3x - 4}{x^2 - x - 6} \, dx = 2 \ln|x + 2| + \ln|x - 3| + C.
\]