Instructions. Your solution to each problem must contain sufficient detail so that it is clear to the reader what your reasoning process is. Full credit will not be given for solutions that are lacking in detail (such as omission of important steps in computing an integral, etc.) State your conclusions and other important steps in your reasoning in complete sentences. You may use a calculator on this exam but you may not use any books or notes.

1. Use L’Hospital’s Rule to show that

\[ \lim_{x \to \infty} x^2 e^{-x} = 0. \]

You must include all details of your reasoning, use correct notation, and write in complete sentences.
2. Show that

\[ \int_{2}^{\infty} x^2 e^{-x} \, dx = \frac{10}{e^2}. \]

You must include all details of your reasoning (steps in computing integrals, etc.)
3. Explain why the series
\[ \sum_{n=2}^{\infty} n^2 e^{-n} \]
converges. In doing this, you may refer to the result of problem 2. You must include all details of your reasoning, use correct notation, and write in complete sentences that do not contain the word “it”.
4. Suppose that $a_n$ and $b_n$ are sequences such that $0 \leq a_n \leq b_n$ for all $n \geq 1$. Decide whether each of the following statements must be true or is not necessarily true. (Circle the correct choice.)

(a) If $\lim_{n \to \infty} b_n = 0$, then $\sum_{n=1}^{\infty} a_n$ converges.  
   (must be true / is not necessarily true).

(b) If $\sum_{n=1}^{\infty} a_n$ converges, then $\sum_{n=1}^{\infty} b_n$ also converges.  
   (must be true / is not necessarily true).

(c) If $\sum_{n=1}^{\infty} b_n$ converges, then $\lim_{n \to \infty} a_n = 0$.  
   (must be true / is not necessarily true).

(d) If $\lim_{n \to \infty} \frac{a_n}{b_n} = 1$, then both $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ must be convergent.  
   (must be true / is not necessarily true).

(e) If $\sum_{n=1}^{\infty} a_n$ converges, then $\sum_{n=1}^{\infty} b_n$ diverges.  
   (must be true / is not necessarily true).
5. Use either the Standard Comparison Test or the Limit Comparison Test to explain why the series
\[ \sum_{n=1}^{\infty} \frac{\sqrt{n}}{n + 1} \]
diverges. You may use the fact that you already know about the convergence–divergence behavior of \( p \) series.

You must include all details of your reasoning, use correct notation, and write in complete sentences that do not contain the word “it”.