1. Use differentiation to check that the formula
\[ \int (x^2 + 1) e^{-x} \, dx = -(x^2 + 2x + 3) e^{-x} + C \]
is correct.

2. Use integration by parts to show how the above formula is obtained.

Solution:

1. 
\[ \frac{d}{dx} \left( -(x^2 + 2x + 3) e^{-x} \right) = -\frac{d}{dx} \left( (x^2 + 2x + 3) e^{-x} \right) \]
which by the Product Rule (and Chain Rule) equals
\[ (x^2 + 2x + 3) \cdot \frac{d}{dx} (e^{-x}) + (e^{-x}) \cdot \frac{d}{dx} (x^2 + 2x + 3) \]
\[ = (2x + 2)(-e^{-x}) + (e^{-x})(x^2 + 2x + 3) \]
\[ = e^{-x}(-2x - 2 + x^2 + 2x + 3) \]
\[ = (x^2 + 1) e^{-x}. \]
This shows that the given formula is correct.

2. To show how to obtain the formula using integration by parts, we let

\[
\begin{array}{c|c}
  u = x^2 + 1 & dv = e^{-x} \, dx \\
  du = 2x \, dx & v = -e^{-x}
\end{array}
\]

This gives us
\[ \int (x^2 + 1) e^{-x} \, dx = \int u \, dv \]
\[ = uv - \int v \, du \]
\[ = -(x^2 + 1) e^{-x} + 2 \int xe^{-x} \, dx. \]
We now have a separate problem to do (which also requires integration by parts).

\[ \int x e^{-x} \, dx =? \]

To do this problem, we let

\[
\begin{array}{|c|c|}
\hline
u &= x \\
\hline
dv &= e^{-x} \, dx \\
\hline
\end{array}
\]

and obtain

\[
\int x e^{-x} \, dx = \int u \, dv = uv - \int v \, du = -xe^{-x} + \int e^{-x} \, dx = -xe^{-x} - e^{-x} + C = -e^{-x} (x + 1) + C.
\]

Substituting this result back into our earlier work gives

\[
\int (x^2 + 1) e^{-x} \, dx = -(x^2 + 1) e^{-x} + 2 \int x e^{-x} \, dx = -(x^2 + 1) e^{-x} + 2 (-e^{-x} (x + 1) + C) = -e^{-x} (x^2 + 1 + 2x + 2) + C = -e^{-x} (x^2 + 2x + 3) + C.
\]