Evaluate the indefinite integral
\[ \int \sin^3(x) \cos^3(x) \, dx. \]

You must include all steps. (Be detailed in showing which trigonometric identities you are using to simplify the integrand. Also be sure to show any substitutions that you are making.)

Solution:
Observe that
\[
\sin^3(x) \cos^3(x) = \sin^3(x) \cos^2(x) \cos(x)
\]
\[ = \sin^3(x) (1 - \sin^2(x)) \cos(x) \]
\[ = (\sin^3(x) - \sin^5(x)) \cos(x). \]

Thus, if we make the simple substitution \( u = \sin(x), \, du = \cos(x) \, dx \), we obtain
\[
\int \sin^3(x) \cos^3(x) \, dx = \int (\sin^3(x) - \sin^5(x)) \cos(x) \, dx
\]
\[ = \int (u^3 - u^5) \, du \]
\[ = \frac{1}{4} u^4 - \frac{1}{6} u^6 + C \]
\[ = \frac{1}{4} \sin^4(x) - \frac{1}{6} \sin^6(x) + C. \]