1. Use differentiation to check that the formula

\[ \int \frac{1}{\sqrt{9-x^2}} \, dx = \arcsin \left( \frac{1}{3} x \right) + C \]

is correct.

2. Show how the method of trigonometric substitution is used to obtain the above formula. (OR This integral can actually be done by using algebra and a simple substitution. You could show how to do it that way instead of using trigonometric substitution.)

Solution:

1. \[
\frac{d}{dx} \left( \arcsin \left( \frac{1}{3} x \right) \right) = \frac{1}{\sqrt{1 - \left( \frac{1}{3} x \right)^2}} \cdot \frac{1}{3} = \frac{1}{\sqrt{9} \cdot \sqrt{1 - \frac{x^2}{9}}} = \frac{1}{\sqrt{9-x^2}}.
\]

This shows that the given formula is correct.

2. To use trigonometric substitution to do this integral, we let

\[ x = 3 \sin (\theta) \]
\[ dx = 3 \cos (\theta) \, d\theta \]

which gives us

\[ \sqrt{9-x^2} = \sqrt{9-9 \sin^2 (\theta)} = \sqrt{9 \left(1 - \sin^2 (\theta)\right)} = \sqrt{9 \cos^2 (\theta)} = 3 \cos (\theta). \]

We now have

\[ \int \frac{1}{\sqrt{9-x^2}} \, dx = \int \frac{1}{3 \cos (\theta)} \cdot 3 \cos (\theta) \, d\theta = \int 1 \, d\theta = \theta + C = \arcsin \left( \frac{1}{3} x \right) + C. \]

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