NAME______________________________________

A tank contains 2000 liters of brine with 15 kg of dissolved salt. Pure water enters the tank at a rate of 20 liters per minute. The solution is kept thoroughly mixed and drains from the tank at the same rate.

1. How much salt is in the tank after 10 minutes?
2. After how long will there be 5 kg of salt in the tank?

**Note:** You must include all details of how you go about answering these questions. It is not sufficient to just write down answers (even if they are correct answers) without explaining exactly how you arrive at your answers.

**Solution:** Let \( A(t) \) be the amount of salt in the tank at time \( t \). Then

\[
\frac{dA}{dt} = \left( \frac{201}{\text{min}} \right) \left( \frac{0 \text{ kg}}{1} \right) - \left( \frac{201}{\text{min}} \right) \left( \frac{A(t) \text{ kg}}{2000 \text{ l}} \right).
\]

Thus \( A \) satisfies

\[
\frac{dA}{dt} = -0.01A
\]

\[ A(0) = 15. \]

The solution to the above initial value problem is

\[ A(t) = 15e^{-0.01t}. \]

Thus the amount of salt in the tank after 10 minute is

\[ A(10) = 15e^{-0.01(10)} \approx 13.57 \text{ kg}. \]

To find the time at which we have 5 kg of salt in the tank, we must solve

\[ 15e^{-0.01t} = 5. \]

The solution of the above equation is

\[ t = -\frac{100 \ln \left( \frac{1}{3} \right)}{10} \approx 109.86. \]

Thus it takes about 110 minutes for the amount of salt in the tank to reach 5 kg.