A bacteria culture initially contains 500 cells and grows at a rate proportional to its size. After 30 minutes the population has increased to 1250 cells.

1. Find an expression for the number of bacteria after \( t \) hours.

2. After how long will there be 5000 cells in the culture?

**Note:** You must include all details of how you go about answering these questions. It is not sufficient to just write down answers (even if they are correct answers) without explaining exactly how you arrive at your answers.

**Solution:** Let \( P(t) \) be the number of cells at time \( t \) hours. Then \( P \) satisfies

\[
\frac{dP}{dt} = kP \\
P(0) = 500.
\]

The solution to the above initial value problem is

\[ P(t) = 500e^{kt}. \]

Since we are told that \( P \left( \frac{1}{2} \right) = 1250 \), we must have

\[ 500e^{\frac{k}{2}} = 1250 \]

and this implies that

\[ e^{\frac{k}{2}} = 2.5 \]

and hence that

\[ e^k = (2.5)^2 = 6.25. \]

Thus the number of cells in the culture at time \( t \) hours is

\[ A(t) = 500(6.25)^t. \]

Another way to write this expression is

\[ A(t) = 500e^{kt} \]

where

\[ k = 2 \ln (2.5) = \ln (6.25) \]
To find the time at which we have 5000 cells, we must solve

$$500e^{kt} = 5000.$$ 

The solution of the above equation is

$$t = \frac{\ln(10)}{\ln(6.25)} \approx 1.26.$$ 

Thus it takes about 1.26 hours for the population of the culture to reach 5000 cells.