NAME_______________________________________________

A cold drink with a temperature of 38°F is removed from a refrigerator. After sitting for 20 minutes in a 74°F room, the temperature of the drink is 50°F. Answer the following questions (assuming that the warming of the drink is governed by Newton’s Law).

1. What is the temperature of the drink 40 minutes after it is removed from the refrigerator?

2. How long does it take for the temperature of the drink to reach 70°C?

**Note:** You must include all details of how you go about answering these questions. It is not sufficient to just write down answers (even if they are correct answers) without explaining exactly how you arrive at your answers.

**Solution:** Let \( T(t) \) be the temperature of the drink at time \( t \) minutes. Then \( T \) satisfies

\[
\frac{dT}{dt} = k(74 - T)
\]

\( T(0) = 38. \)

The solution to the above initial value problem is

\[ T(t) = 74 - 36e^{-kt}. \]

Since we are told that \( T(20) = 50 \), we must have

\[ 74 - 36e^{-20k} = 50 \]

and this implies that

\[ (e^{-k})^{20} = \frac{2}{3} \]

and hence that

\[ e^{-k} = \left( \frac{2}{3} \right)^{1/20}. \]

Thus the temperature of the drink at time \( t \) minutes is

\[ T(t) = 74 - 36 \left( \frac{2}{3} \right)^{\frac{1}{20}t}. \]

Another way to write this expression is

\[ T(t) = 74 - 36e^{-kt} \]
where
\[ k = -\frac{1}{20} \ln \left( \frac{2}{3} \right). \]

The temperature of the drink 40 minutes after it is removed from the refrigerator is
\[ T(40) = 74 - 36 \left( \frac{2}{3} \right)^{\frac{1}{40}} = 58^\circ C. \]

To find the time at which the temperature of the drink is 70$^\circ C$, we must solve
\[ 74 - 36e^{-kt} = 70. \]

The solution of the above equation is
\[ t = \frac{\ln(9)}{k} \approx 108.38. \]

Thus it takes about 108 minutes for the temperature of the drink to reach 70$^\circ C$.