Instructions. This exam contains four indefinite integrals (problems 1–4) and one definite integral (problem 5) with answers given. For each indefinite integral you must show a) how to evaluate the integral (using the strategies we have been studying such as basic substitution, integration by parts, trigonometric substitution, etc.) and b) how to check that the answer you get is correct (by differentiation). For problem 5, you must show the details of how integration and the Fundamental Theorem of Calculus (Part 2) is used to obtain the answer. Here is an example of how your work on problems 1–4 should appear:

Example:

\[ \int x \sec^2 (x^2) \, dx = \frac{1}{2} \tan (x^2) + C. \]

a) To evaluate the above integral, we use a basic substitution: Let \( u = x^2 \). Then \( du = 2x \, dx \) and we have

\[ \int x \sec^2 (x^2) \, dx = \frac{1}{2} \int \sec^2 (u) \, du = \frac{1}{2} \tan (u) + C = \frac{1}{2} \tan (x^2) + C. \]

b) To check that this result is correct, we observe that

\[ \frac{d}{dx} \left( \frac{1}{2} \tan (x^2) \right) = \frac{1}{2} \sec^2 (x^2) \cdot \frac{d}{dx} (x^2) = \frac{1}{2} \sec^2 (x^2) \cdot 2x = x \sec^2 (x^2). \]

Some of the following trigonometric identities will be needed for certain problems:

\[
\begin{array}{|c|c|}
\hline
\sin^2 (\theta) + \cos^2 (\theta) = 1 & \sin (\theta) \cos (\theta) = \frac{1}{2} \sin (2\theta) \\
\tan^2 (\theta) + 1 = \sec^2 (\theta) & \cos^2 (\theta) = \frac{1}{2} (1 + \cos (2\theta)) \\
1 + \cot^2 (\theta) = \csc^2 (\theta) & \sin^2 (\theta) = \frac{1}{2} (1 - \cos (2\theta)) \\
\hline
\end{array}
\]

1.

\[ \int x^2 \left( \frac{x^3}{18} - 1 \right)^5 \, dx = \left( \frac{x^3}{18} - 1 \right)^6 + C \]

2.

\[ \int x^2 e^{4x} \, dx = \frac{1}{32} e^{4x} (8x^2 - 4x + 1) + C \]

3.

\[ \int \frac{1}{\sqrt{9 + x^2}} \, dx = \ln \left( \frac{1}{3} \sqrt{9 + x^2} + \frac{1}{3} x \right) + C. \]

4.

\[ \int \frac{16x^3}{4x^2 - 4x + 1} \, dx = 2x^2 + 4x + 3 \ln (|2x - 1|) - \frac{1}{2x - 1} + C. \]

Hint: \( 4x^2 - 4x + 1 = (2x - 1)^2 \)

5.

\[ \int_0^{\pi/6} 3 \cos^5 (3x) \, dx = \frac{8}{15} \]