When the region pictured in Figure A is revolved about the $x$ axis, we obtain the solid pictured in Figure B (a cone with a curved interior).

\[ y = (x - 1)^4 \]

\[ y = x - 1 \]

Figure A

Figure B

a) Use the washer method to obtain a definite integral that gives the volume of this solid.
b) Use the shell method to obtain a definite integral that gives the volume of this solid.

c) Evaluate one of the integrals that you obtained in order to find the volume of the solid.

Solution:

a) Washer Method

\[ V = \pi \int_{1}^{2} (y_{a}^{2} - y_{l}^{2}) \, dx \]
\[ = \pi \int_{1}^{2} ((x - 1)^{2} - (x - 1)^{8}) \, dx \]
\[ = \pi \int_{0}^{1} (u^{2} - u^{8}) \, dx \]
\[ = \pi \left( \frac{1}{3}u^{3} - \frac{1}{9}u^{9} \right) \bigg|_{u=1}^{u=0} \]
\[ = \frac{2\pi}{9}. \]

b) Shell Method

\[ V = 2\pi \int_{0}^{1} y (x_{r} - x_{l}) \, dy \]
\[ = 2\pi \int_{0}^{1} y \left( \left( y^{1/4} + 1 \right) - (y + 1) \right) \, dy \]
\[ = 2\pi \int_{0}^{1} \left( y^{5/4} - y^{2} \right) \, dy \]
\[ = 2\pi \left( \frac{4}{9} y^{9/4} - \frac{1}{3} y^{3} \right) \bigg|_{y=1}^{y=0} \]
\[ = \frac{2\pi}{9}. \]