NAME___________________

There are two problems on this quiz. Each problem will be graded separately and given a score of 0, 4, 10, 16, or 20. The two scores will then be averaged to obtain the overall quiz score. Please write neatly and clearly.

1) Find the general solution of the differential equation
\[ \frac{dy}{dx} = e^{x+y}. \]
You must include all steps in the solution process.

**Solution:** Note that \( e^{x+y} = e^x e^y \) so we can write the differential equation as
\[ \frac{dy}{dx} = e^x e^y. \]
Using separation of variables we obtain
\[ \int e^{-y} \frac{dy}{dx} \, dx = \int e^x \, dx \]
which is the same as
\[ \int e^{-y} \, dy = \int e^x \, dx. \]
By performing these integrations, we obtain
\[ -e^{-y} = e^x + C \]
which gives
\[ e^{-y} = -e^x + C \]
which gives
\[ -y = \ln(-e^{-x} + C) \]
which gives
\[ y = -\ln(-e^{-x} + C) \]

**Conclusion:** The general solution of the given differential equation is
\[ y = -\ln(-e^{-x} + C). \]

2) The intensity, \( L \), of light \( x \) feet under the surface of the ocean satisfies the differential equation
\[ \frac{dL}{dx} = -kL. \]
As a diver, you know from experience that diving to 18 feet in the Caribbean Sea cuts the light intensity in half. You cannot work without artificial light when the intensity falls below one tenth of the surface value. About how deep can you expect to work without artificial light?

(Note: The answer is about 59.8 feet. You must show how this answer is obtained.)
**Solution:** The initial value problem that models this situation is
\[
\frac{dy}{dt} = -kL
\]
\[y|_{x=0} = y_0.\]
We know that the solution of this initial value problem is
\[y = y_0 e^{-kt}.\]
Also, since
\[y|_{x=18} = \frac{1}{2} y_0,
\]
we have
\[y_0 e^{-18k} = \frac{1}{2} y_0
\]
which gives
\[k = \frac{\ln(2)}{18}.
\]
To find the depth, \(D\), at which \(y|_{x=D} = \frac{1}{10} y_0\), we solve
\[y_0 e^{-kD} = \frac{1}{10} y_0
\]
to obtain
\[D = -\frac{1}{k} \ln\left(\frac{1}{10}\right) = \frac{18 \ln(10)}{\ln(2)} \approx 59.8.
\]
**Conclusion:** The maximum depth at which you could work without artificial light is about 59.8 feet.