1) The series
\[ \sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{n^2} = 1 - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \cdots \]
is absolutely convergent (and hence convergent) because the series
\[ \sum_{n=1}^{\infty} \left| (-1)^{n+1} \frac{1}{n^2} \right| = 1 + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \cdots \]
is a convergent (because it is a \( p \) series with \( p = 2 \)).

3) The series
\[ \sum_{n=1}^{\infty} (-1)^{n+1} \left( \frac{n}{10} \right)^n \]
is divergent. This is because the series
\[ \sum_{n=1}^{\infty} \left| (-1)^{n+1} \left( \frac{n}{10} \right)^n \right| = \sum_{n=1}^{\infty} \left( \frac{n}{10} \right)^n \]
diverges by the Root Test (and hence also by the Basic Divergence Test) because
\[ \lim_{n \to \infty} \sqrt[n]{\left( \frac{n}{10} \right)^n} = \lim_{n \to \infty} \frac{n}{10} = \infty. \]

5) The series
\[ \sum_{n=2}^{\infty} (-1)^{n+1} \frac{1}{\ln(n)} \]
converges by the Alternating Series Test. This is because:

a) \( u_n = \frac{1}{\ln(n)} > 0 \) for all \( n \geq 2 \).

b) \( \ln (n + 1) > \ln (n) \) for all \( n \) and hence \( u_{n+1} < u_n \) for all \( n \).

c) \( \lim_{n \to \infty} u_n = 0 \) (because \( \lim_{n \to \infty} \ln (n) = \infty \)).
21) The series
\[ \sum_{n=1}^{\infty} (-1)^{n+1} \frac{1+n}{n^2} \]
converges by the Alternating Series Test. To see why, first note that
\[ u_n = \frac{1+n}{n^2} > 0 \]
for all \( n \geq 1 \) and also observe that
\[ \lim_{n \to \infty} u_n = \lim_{n \to \infty} \frac{1+n}{n^2} = \lim_{n \to \infty} \frac{1/n + 1}{1} = 0 = 0. \]
The only thing that remains is to show that \( u_n \) is monotone decreasing. To do this we observe that
\[ u_n = \frac{1+n}{n^2} = \frac{1}{n^2} + \frac{1}{n}. \]
Since both \( 1/n^2 \) and \( 1/n \) are monotone decreasing, then so is \( u_n \) (because the sum of two monotone decreasing sequences is monotone decreasing).

23) The series
\[ \sum_{n=1}^{\infty} (-1)^n n^2 \left( \frac{2}{3} \right)^n \]
is absolutely convergent (and hence convergent). This is because the Ratio Test can be used to show that the series
\[ \sum_{n=1}^{\infty} \left| (-1)^n n^2 \left( \frac{2}{3} \right)^n \right| = \sum_{n=1}^{\infty} n^2 \left( \frac{2}{3} \right)^n \]
is convergent. (When we apply the Ratio Test, we obtain \( \rho = 2/3 \).)

37) Consider the series
\[ \sum_{n=1}^{\infty} (-1)^n \frac{(2n)!}{2^n n! n}. \]
Let us test this series for absolute convergence. For
\[ a_n = \frac{(2n)!}{2^n n! n}, \]
2
we have
\[ a_{n+1} = \frac{(2n + 2)!}{2^{n+1} (n + 1)! (n + 1)} \]

and
\[
\frac{a_{n+1}}{a_n} = \frac{(2n + 2)!}{2^{n+1} (n + 1)! (n + 1)} \cdot \frac{2^n n! n}{(2n)! (n + 1)!} \\
= \frac{(2n + 2)!}{(2n)!} \cdot \frac{2^n n!}{2^{n+1} (n + 1)!} \cdot \frac{n}{n + 1} \\
= \frac{(2n + 1)(2n + 2)}{2} \cdot \frac{1}{n + 1} \cdot \frac{1}{n + 1} \\
= \frac{1}{2} \cdot \frac{(2n + 1)(2n + 2)n}{(n + 1)(n + 1)}
\]

from which we can see that
\[
\lim_{n \to \infty} \frac{a_{n+1}}{a_n} = \infty
\]

and hence that
\[
\sum_{n=1}^{\infty} \frac{(2n)!}{2^n n! n}
\]
diverges by the Ratio Test. Therefore the series that we are studying is not absolutely convergent. However, we have actually also discovered that the series in question is not convergent. This is because the Ratio Test was used to show that \( \sum_{n=1}^{\infty} a_n \) diverges. Recall that when series is shown diverge by the Ratio Test, then it is also true that that series diverges by the Basic Divergence Test. That is, \( \lim_{n \to \infty} a_n \neq 0 \). Since \( \lim_{n \to \infty} a_n \neq 0 \), then it must also be true that \( \lim_{n \to \infty} (-1)^n a_n \neq 0 \). Therefore
\[
\sum_{n=1}^{\infty} (-1)^n a_n = \sum_{n=1}^{\infty} (-1)^n \frac{(2n)!}{2^n n! n}
\]
diverges by the Basic Divergence Test.

41) Consider the series
\[
\sum_{n=1}^{\infty} (-1)^n \left( \sqrt{n + \sqrt{n}} - \sqrt{n} \right).
\]
For $u_n = \sqrt{n} + \sqrt{n} - \sqrt{n}$, observe that 
\[ n + \sqrt{n} > n \]
for all $n \geq 1$ and hence (by taking the square root of both sides of this inequality) 
\[ \sqrt{n + \sqrt{n}} > \sqrt{n} \]
for all $n \geq 1$. Therefore
\[ u_n = \sqrt{n + \sqrt{n} - \sqrt{n}} > 0 \]
for all $n \geq 1$.

Now let us consider $\lim_{n \to \infty} u_n$. This limit is of the indeterminate form $\infty - \infty$. To evaluate it we use the trick of "rationalizing the numerator" and some additional algebraic simplification:

\[
\lim_{n \to \infty} u_n = \lim_{n \to \infty} \frac{\sqrt{n + \sqrt{n} - \sqrt{n}}}{1} \cdot \frac{\sqrt{n + \sqrt{n} + \sqrt{n}}}{\sqrt{n + \sqrt{n} + \sqrt{n}}}
\]
\[
= \lim_{n \to \infty} \frac{(n + \sqrt{n}) - n}{\sqrt{n + \sqrt{n} + \sqrt{n}}}
\]
\[
= \lim_{n \to \infty} \frac{\sqrt{n}}{\sqrt{n + \sqrt{n} + \sqrt{n}}}
\]
\[
= \lim_{n \to \infty} \frac{\frac{\sqrt{n}}{\sqrt{n + \sqrt{n} + \sqrt{n}}}}{\frac{1}{\sqrt{n}}}
\]
\[
= \lim_{n \to \infty} \frac{1}{\sqrt{1 + \frac{1}{\sqrt{n}}} + 1}
\]
\[
= \frac{1}{\sqrt{1 + 0} + 1}
\]
\[
= \frac{1}{2}.
\]

We can stop at this point. The fact that $\lim_{n \to \infty} u_n \neq 0$ tells us that the series in question diverges by the Basic Divergence Test.