For the function
\[ f(x, y) = x^2 + 3xy - e^x \cos(y), \]
verify that \( f_{xy} = f_{yx} \). What is the name of the theorem that guarantees that \( f_{xy} \) must be equal to \( f_{yx} \)?

**Solution:**

\[ f_x = 2x + 3y - e^x \cos(y) \]

and
\[ f_y = 3x + e^x \sin(y) \]

so
\[ f_{xy} = 3 + e^x \sin(y) \]

and
\[ f_{yx} = 3 + e^x \sin(y). \]

The theorem that guarantees that \( f_{xy} \) must be equal to \( f_{yx} \) (at least when \( f_{xy} \) and \( f_{yx} \) are both continuous functions) is called Clairaut’s Theorem.