Consider the function

\[ f(x, y) = -2x + 2. \]

1. What kind of surface is the graph of this function? Draw the graph.

2. Find the directional derivative of \( f \) at the point \((1, 0)\) in the direction of the vector \( \mathbf{v} = -3i + 4j \).

3. Find the direction in which \( f \) is increasing most rapidly at the point \((1, 0)\).

4. Find the rate of increase of \( f \) in the direction in which \( f \) is increasing most rapidly at the point \((1, 0)\).

**Solution:** The graph of \( f \) is a plane. This graph is shown below.
Graph of \( f(x, y) = -2x + 2 \)

A unit vector in the direction of the vector \( \mathbf{v} = -3\mathbf{i} + 4\mathbf{j} \) is

\[
\mathbf{u} = \frac{1}{|\mathbf{v}|} \mathbf{v} = -\frac{3}{5} \mathbf{i} + \frac{4}{5} \mathbf{j}
\]

and

\[
\nabla f(x, y) = \frac{\partial f}{\partial x} \mathbf{i} + \frac{\partial f}{\partial y} \mathbf{j} = -2\mathbf{i} + 0\mathbf{j} = -2\mathbf{i}
\]

which means that

\[
\nabla f(1, 0) = -2\mathbf{i}.
\]

(Note that \( \nabla f(x, y) \) is the same at any point since the graph is a plane.) The directional derivative of \( f \) at the point \( (1, 0) \) in the direction of the vector \( \mathbf{v} = -3\mathbf{i} + 4\mathbf{j} \) is thus

\[
D_u f(1, 0) = \mathbf{u} \cdot \nabla f(1, 0) = \left( -\frac{3}{5} \mathbf{i} + \frac{4}{5} \mathbf{j} \right) \cdot (-2\mathbf{i}) = \frac{6}{5}.
\]
The maximum rate of change always occurs in the direction of the gradient vector. Thus, the maximum rate of change of \( f \) occurs in the direction of the vector \(-i\). The rate of change in this direction is \( |\nabla f (1, 0)| = |{-2i}| = 2\).