NAME ________________________________

Explain why

\[ \lim_{(x,y) \to (0,0)} \frac{(x^2 + y^2)^3}{4x^2y^2} \]

does not exist. (Hint: Find two different paths of approach to (0, 0) for which different limits are obtained.)

Solution: If we approach along the path \( y = x \), we obtain

\[
\lim_{(x,y) \to (0,0)} \frac{(x^2 + y^2)^3}{4x^2y^2} = \lim_{x \to 0} \frac{(x^2 + x^2)^3}{4x^2x^2} = \lim_{x \to 0} \frac{8x^6}{4x^4} = \lim_{x \to 0} 2x^2 = 0
\]

If we approach along the path \( y = x^2 \), we obtain

\[
\lim_{(x,y) \to (0,0)} \frac{(x^2 + y^2)^3}{4x^2y^2} = \lim_{x \to 0} \frac{(x^2 + x^4)^3}{4x^2x^4} = \lim_{x \to 0} \frac{(x^2 + x^4)^3}{4x^6} = \lim_{x \to 0} \frac{1}{4} \left( \frac{x^2 + x^4}{x^2} \right)^3 = \lim_{x \to 0} \frac{1}{4} \left( 1 + x^2 \right)^3 = \frac{1}{4}.
\]

Therefore the limit in question does not exist.